

Improving quantum communication rates with permutation-invariant codes

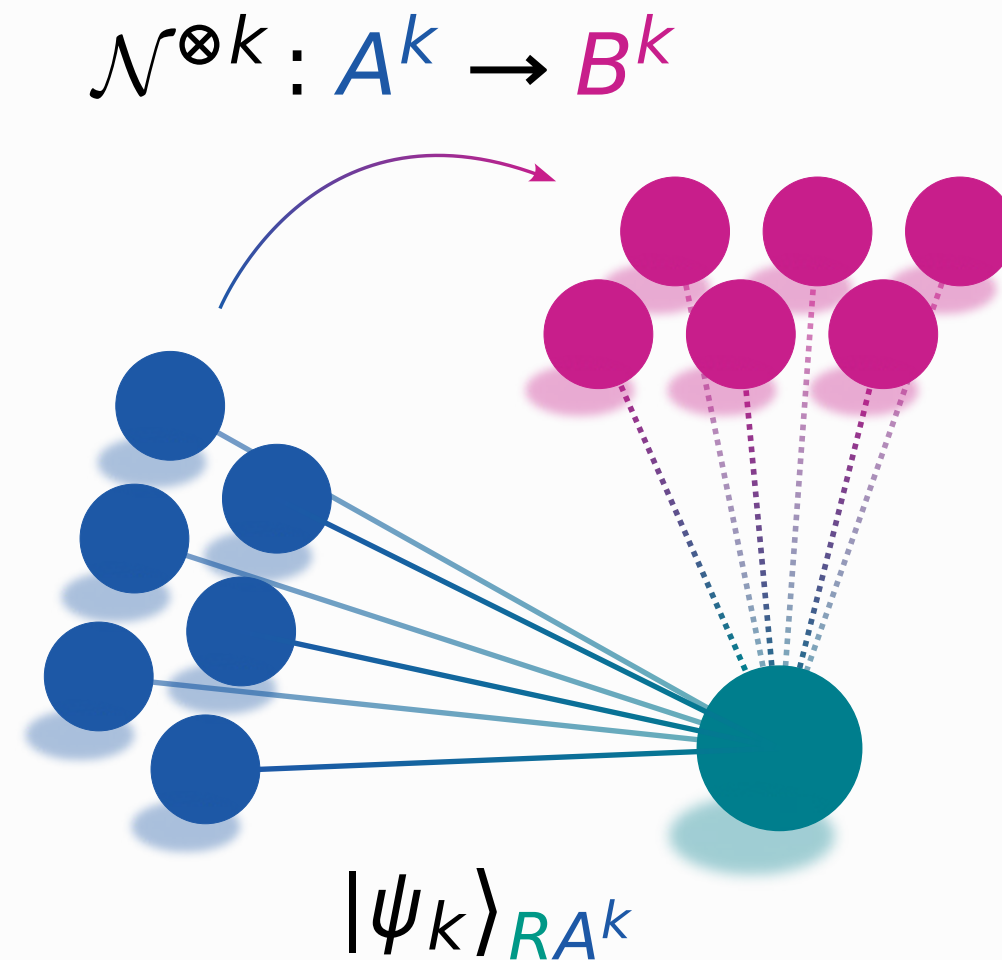
arXiv:2508.09978

Felix Leditzky

Department of Mathematics and IQUIST
University of Illinois Urbana-Champaign



Joint work with my student
Sujeet Bhalerao (UIUC Math)



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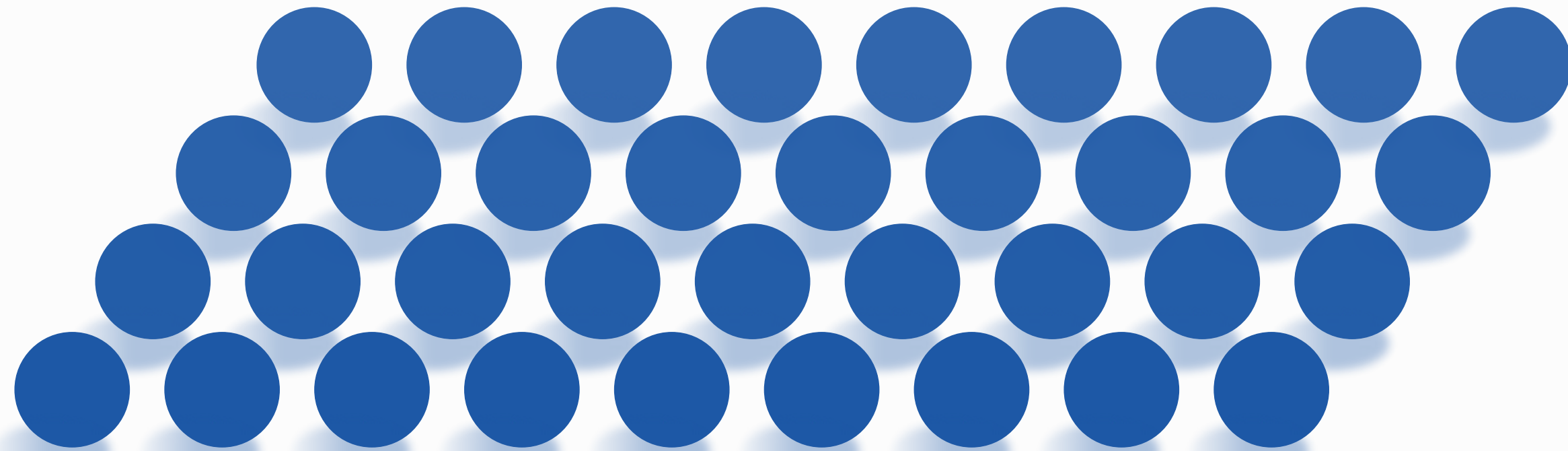
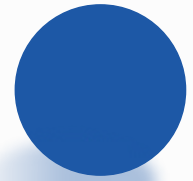


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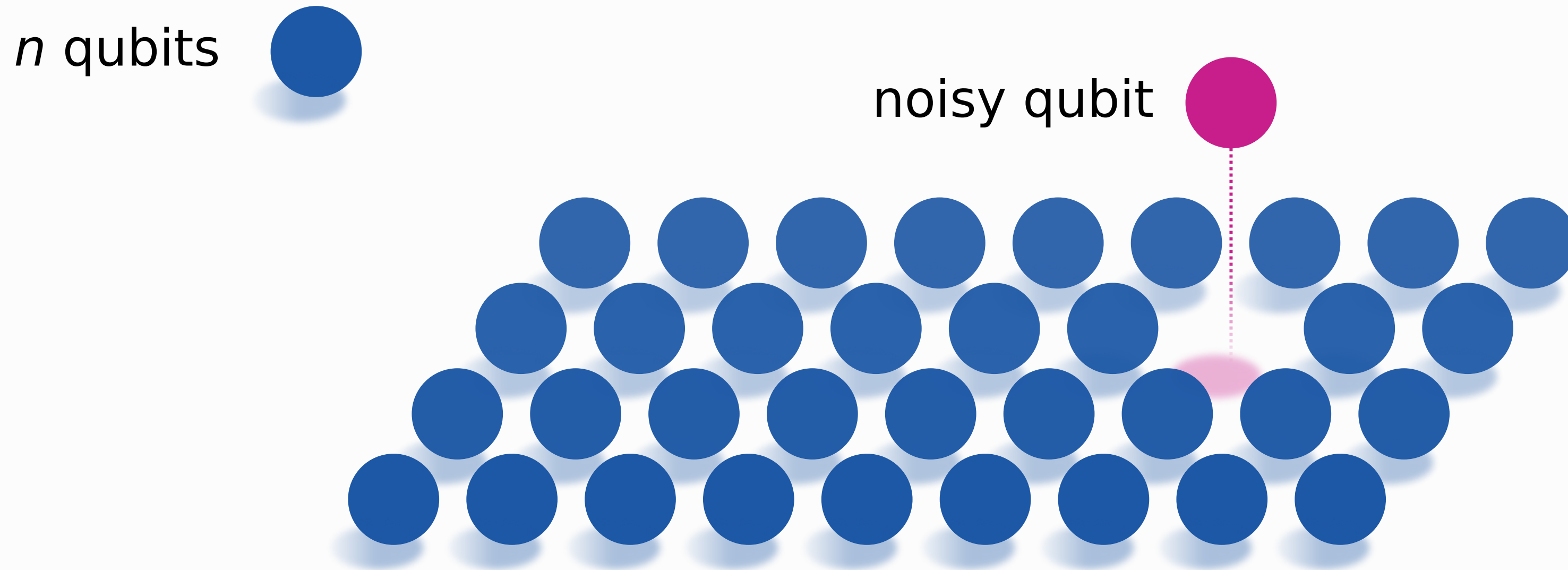
Noise in quantum devices



n qubits



Noise in quantum devices

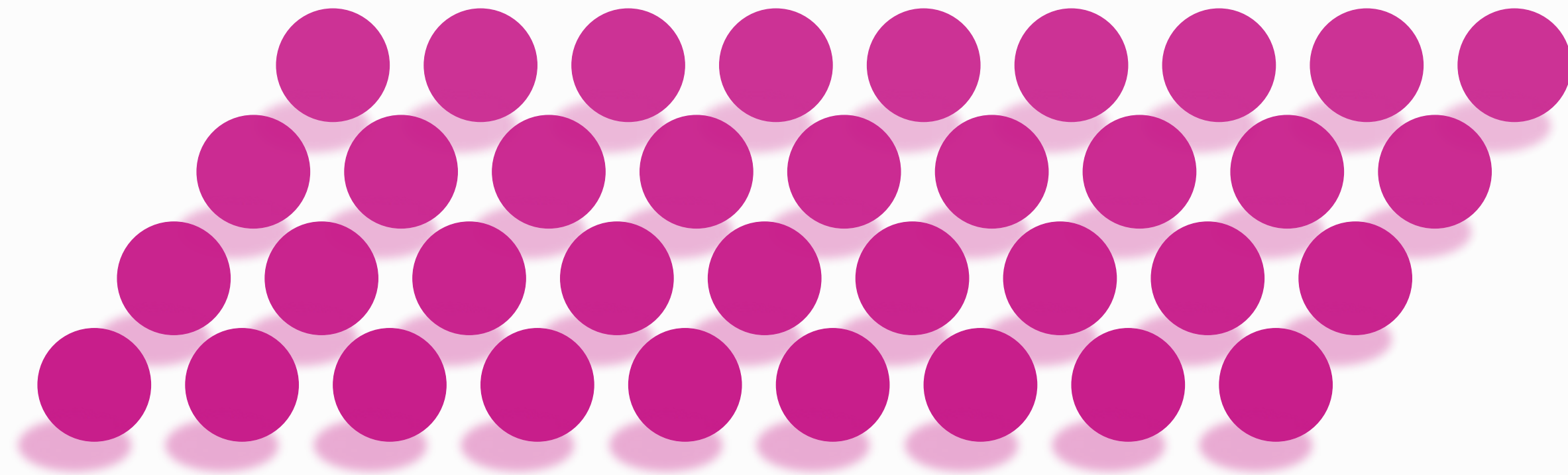


$$\text{pink circle} = \mathcal{N}(\text{blue circle}) = (1 - p) \text{blue circle} + p [q_1 X \text{blue circle} X + q_2 Y \text{blue circle} Y + q_3 Z \text{blue circle} Z]$$

Noise in quantum devices



n noisy qubits $\bullet = \mathcal{N}(\bullet) = (1-p)\bullet + p[q_1 X\bullet X + q_2 Y\bullet Y + q_3 Z\bullet Z]$

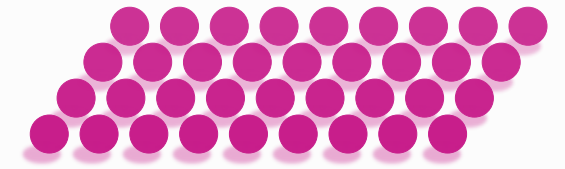


Independent and identically distributed (i.i.d.) noise model $\mathcal{N}^{\otimes n}$

Noise in quantum devices



How much quantum information can be stored in a device

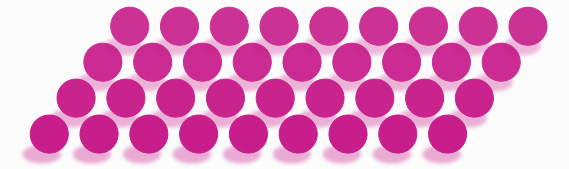


?

Noise in quantum devices

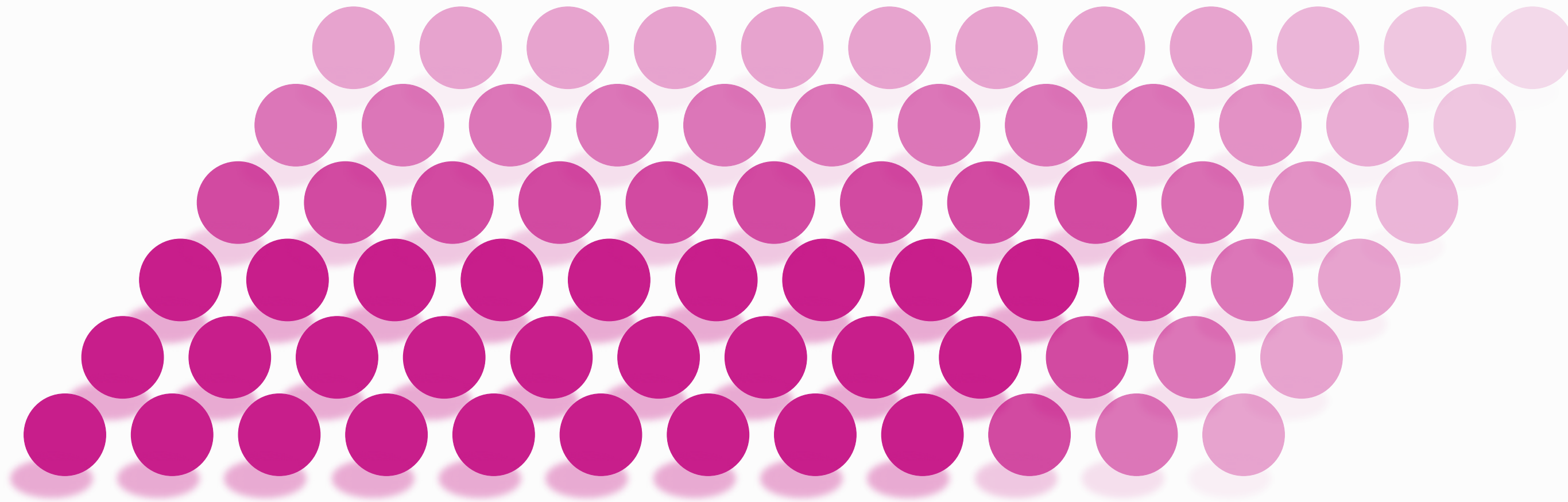


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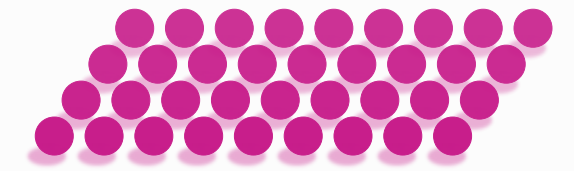
Step 1: Extend n -qubit system infinitely ($n \rightarrow \infty$).



Noise in quantum devices

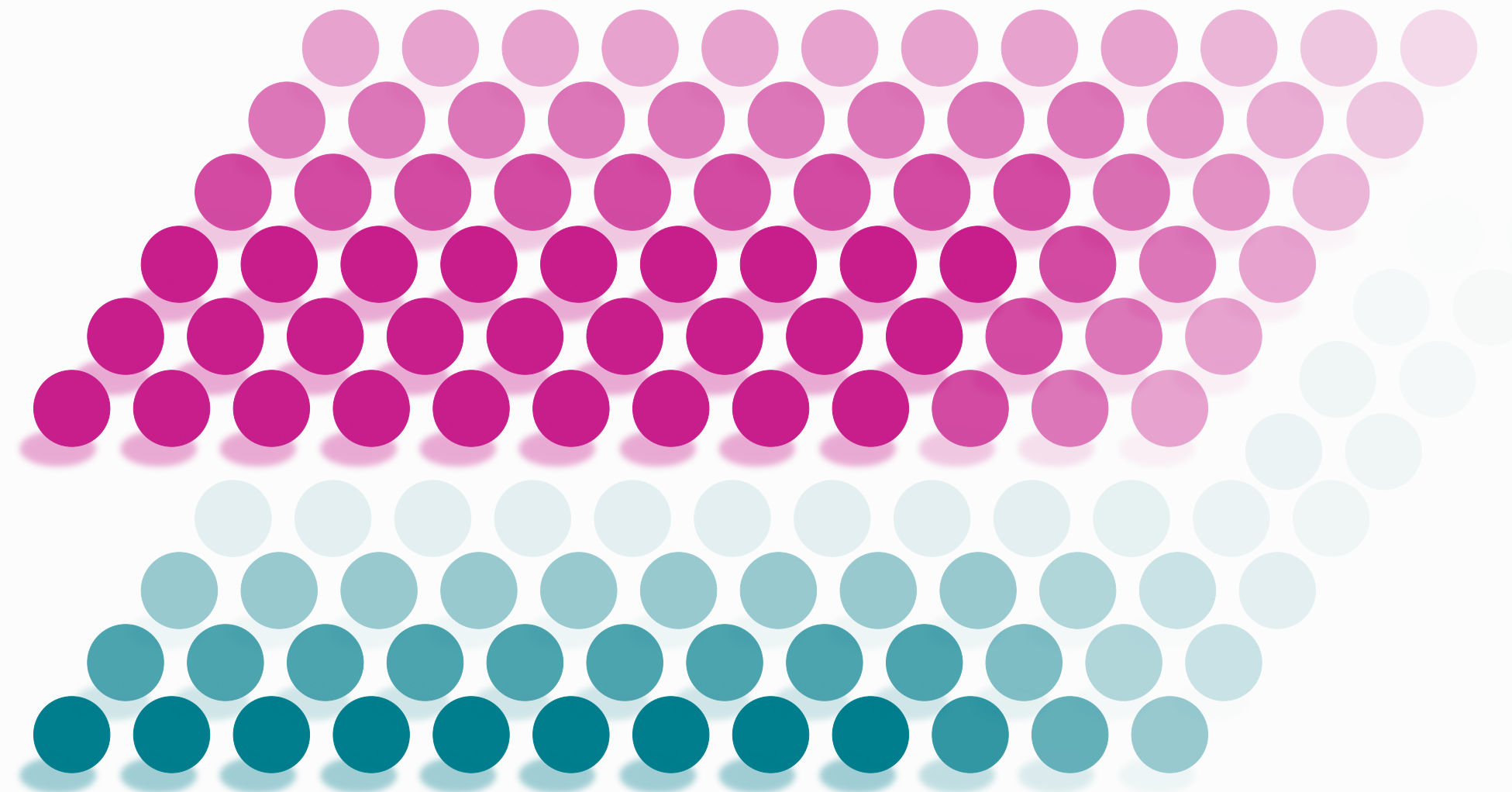


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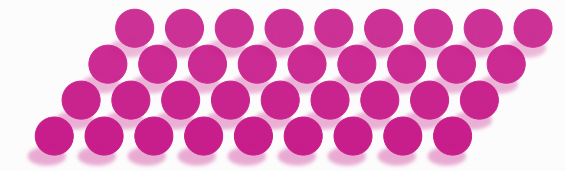
Step 2: Consider a **reference qubit** for each **system qubit**.



Noise in quantum devices

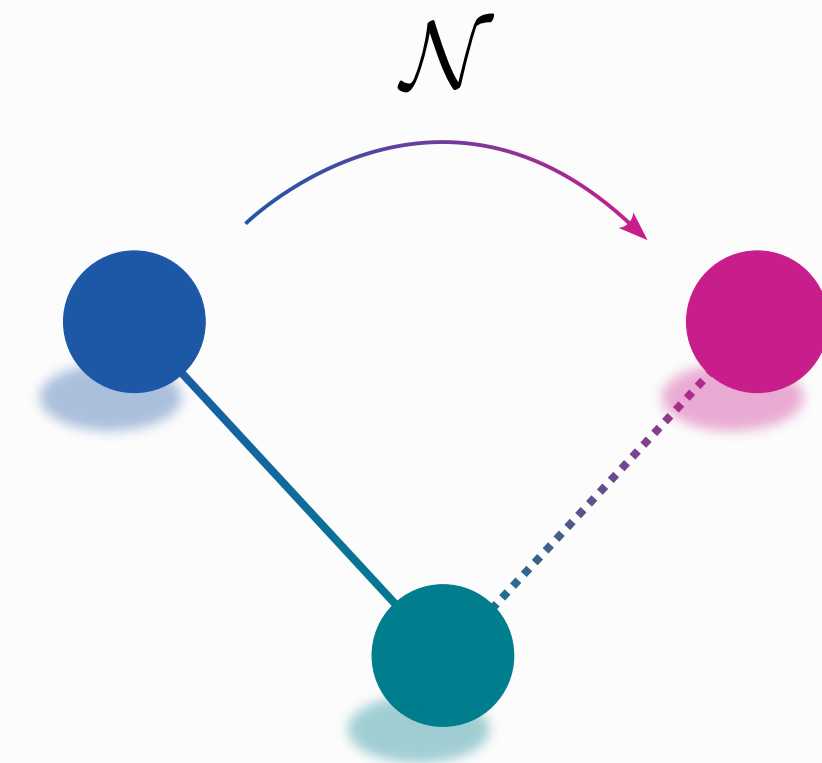
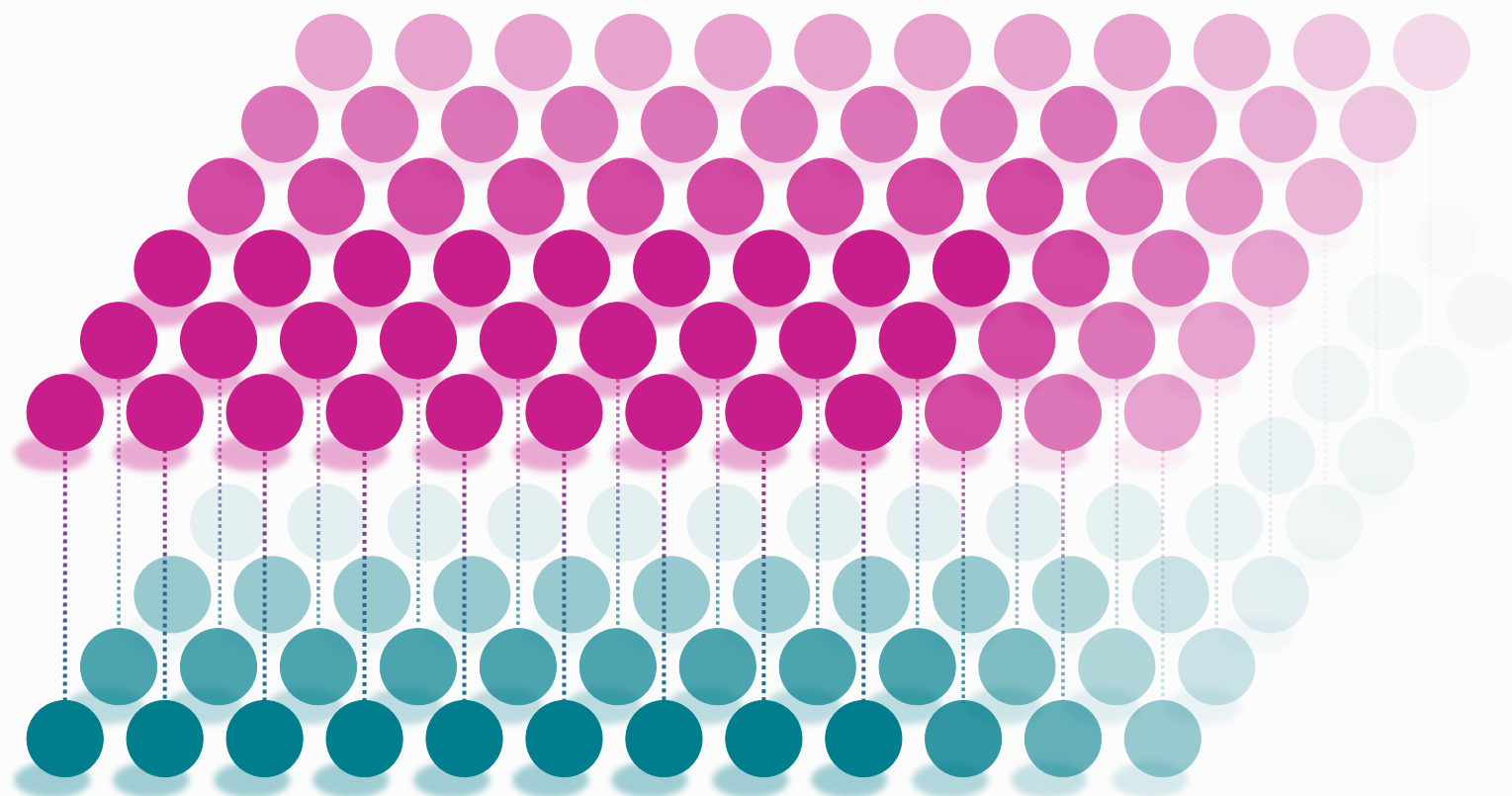


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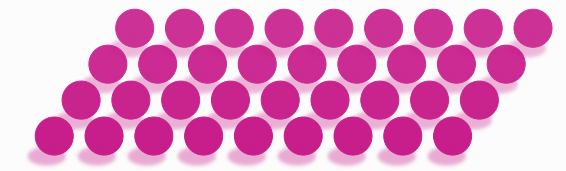
Step 3: Quantify how much initial correlation between **system** and **reference qubits** survives the noise.



Noise in quantum devices

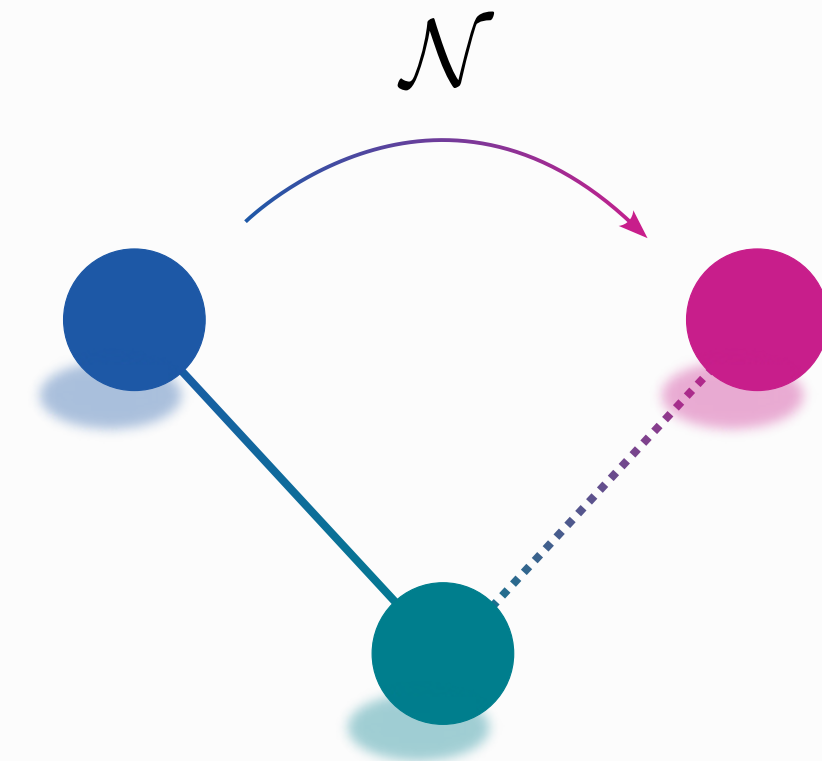
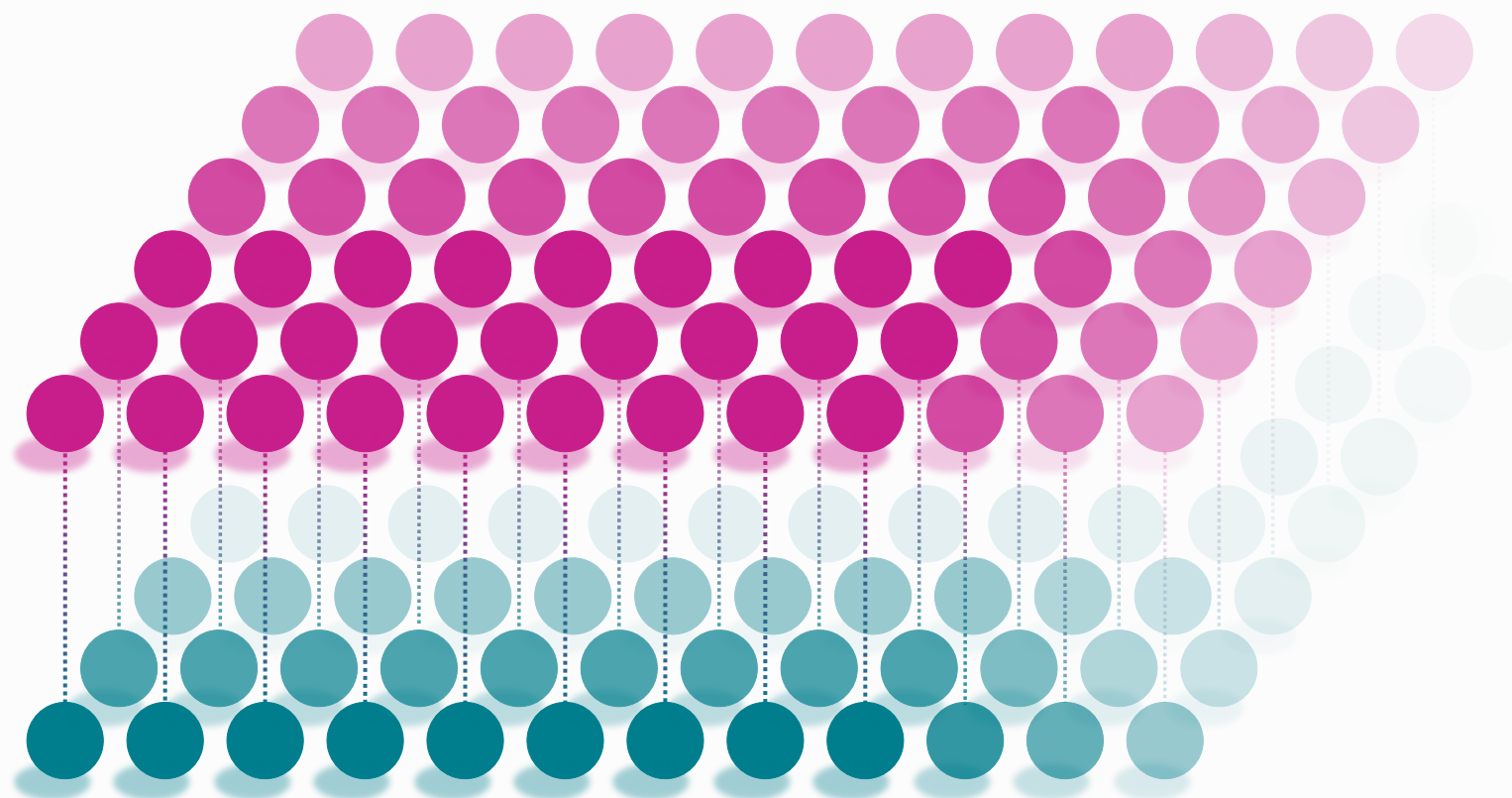


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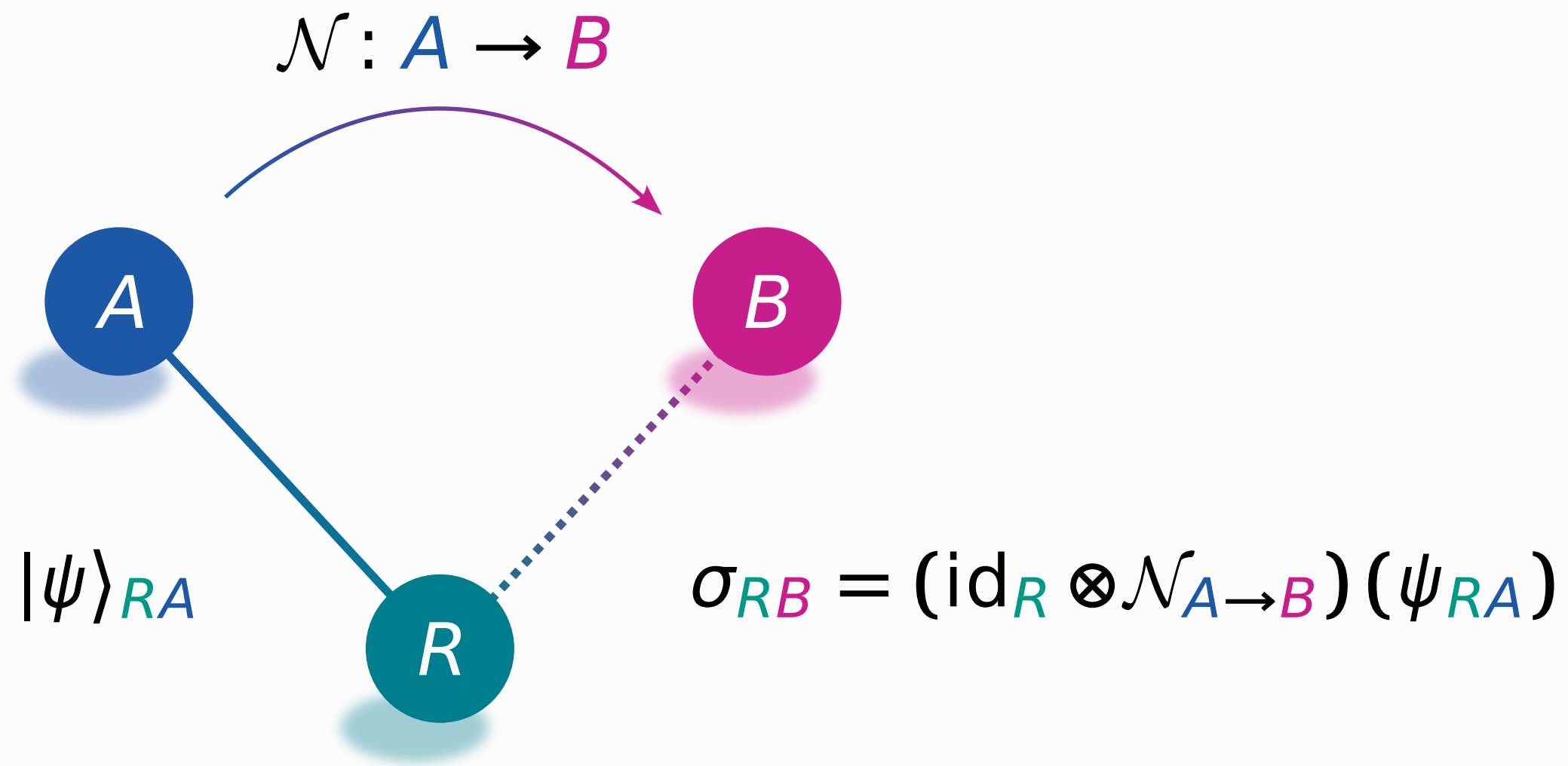
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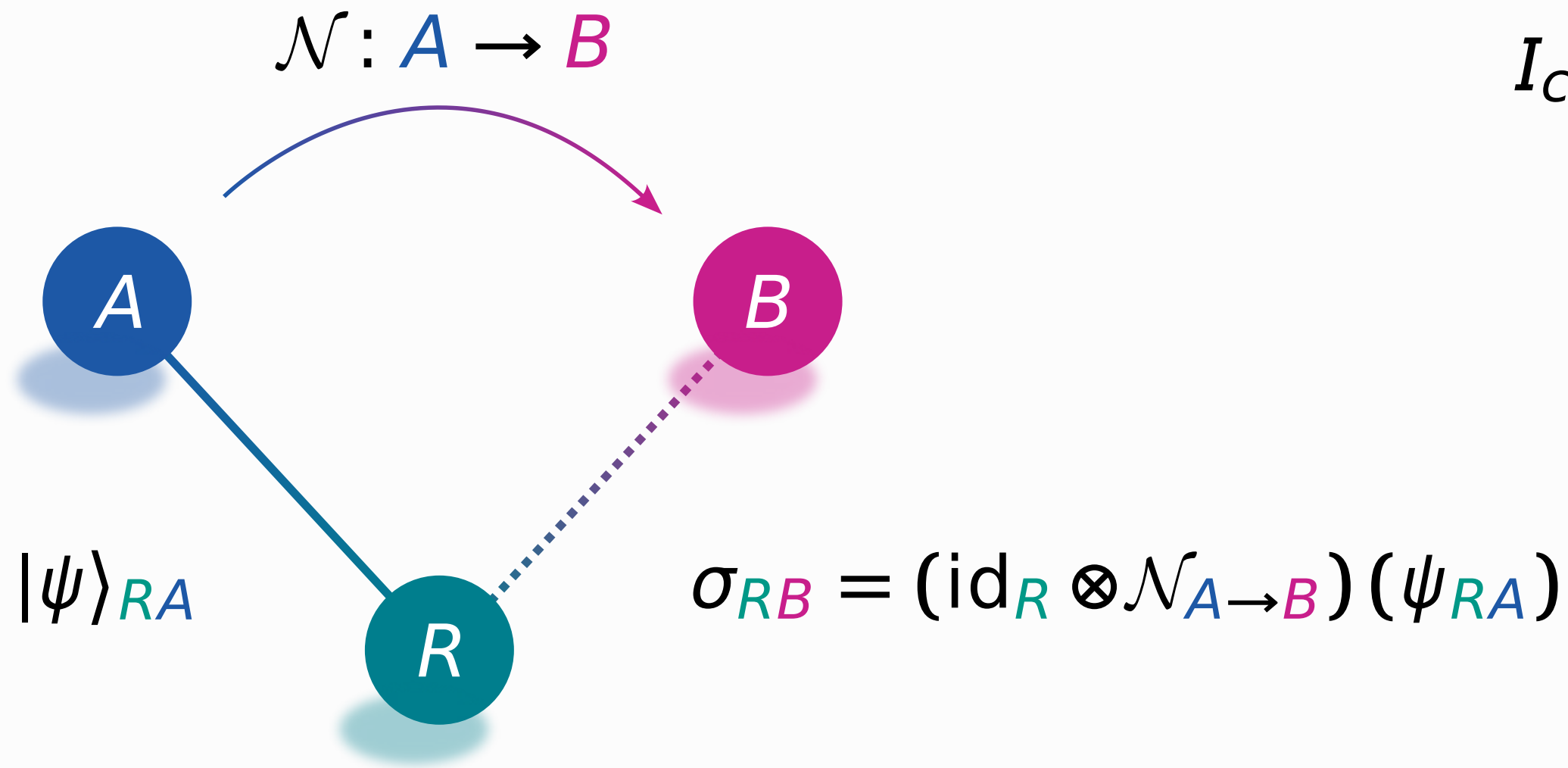


How to quantify correlations?

Quantifying quantum correlations



Quantifying quantum correlations



Coherent information

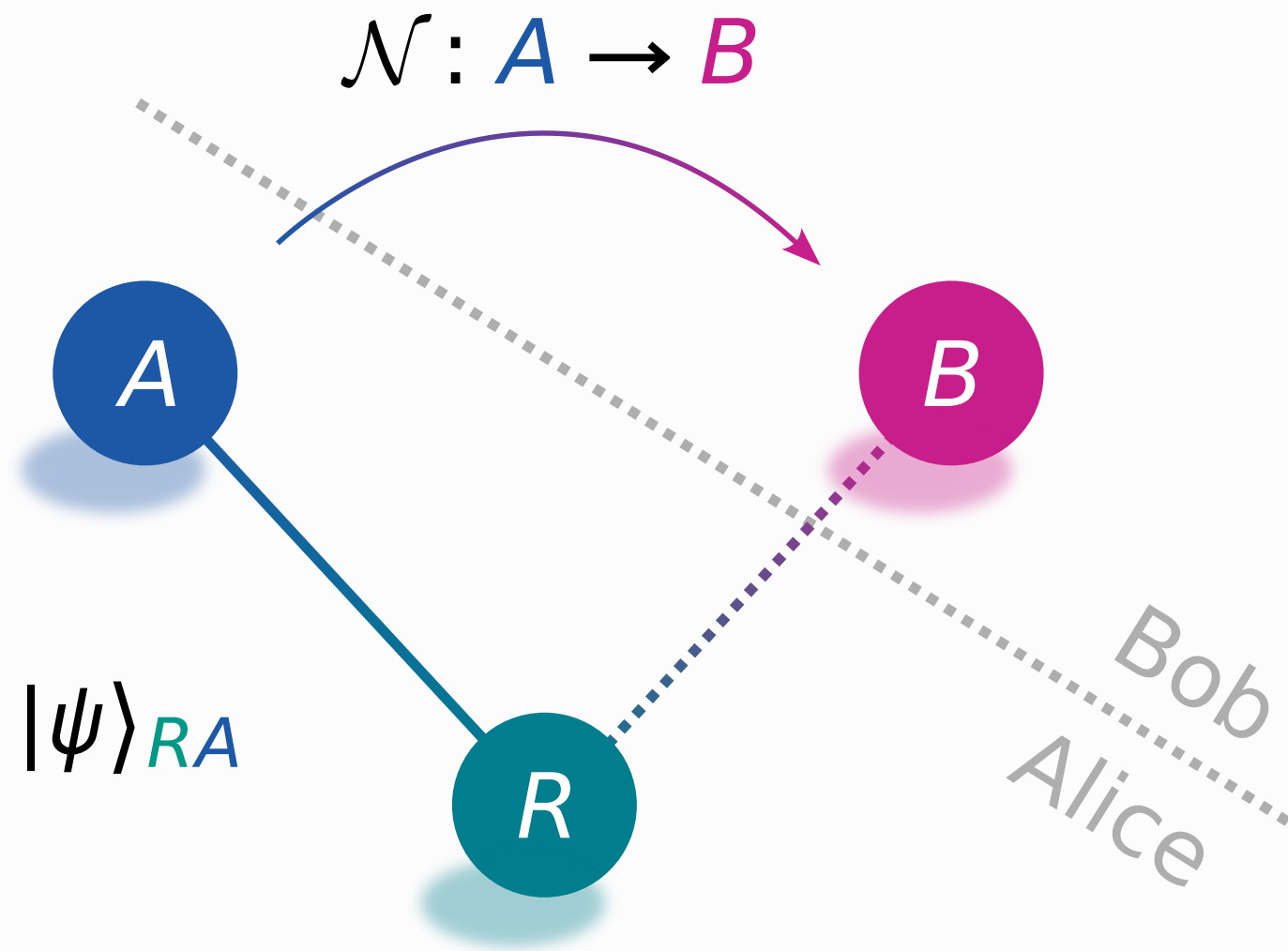
$$I_c(\mathcal{N}, \psi) = S(B)_\sigma - S(RB)_\sigma$$

$$S(Q)_\sigma = -\text{tr}(\sigma_Q \log \sigma_Q)$$

Quantifying quantum correlations



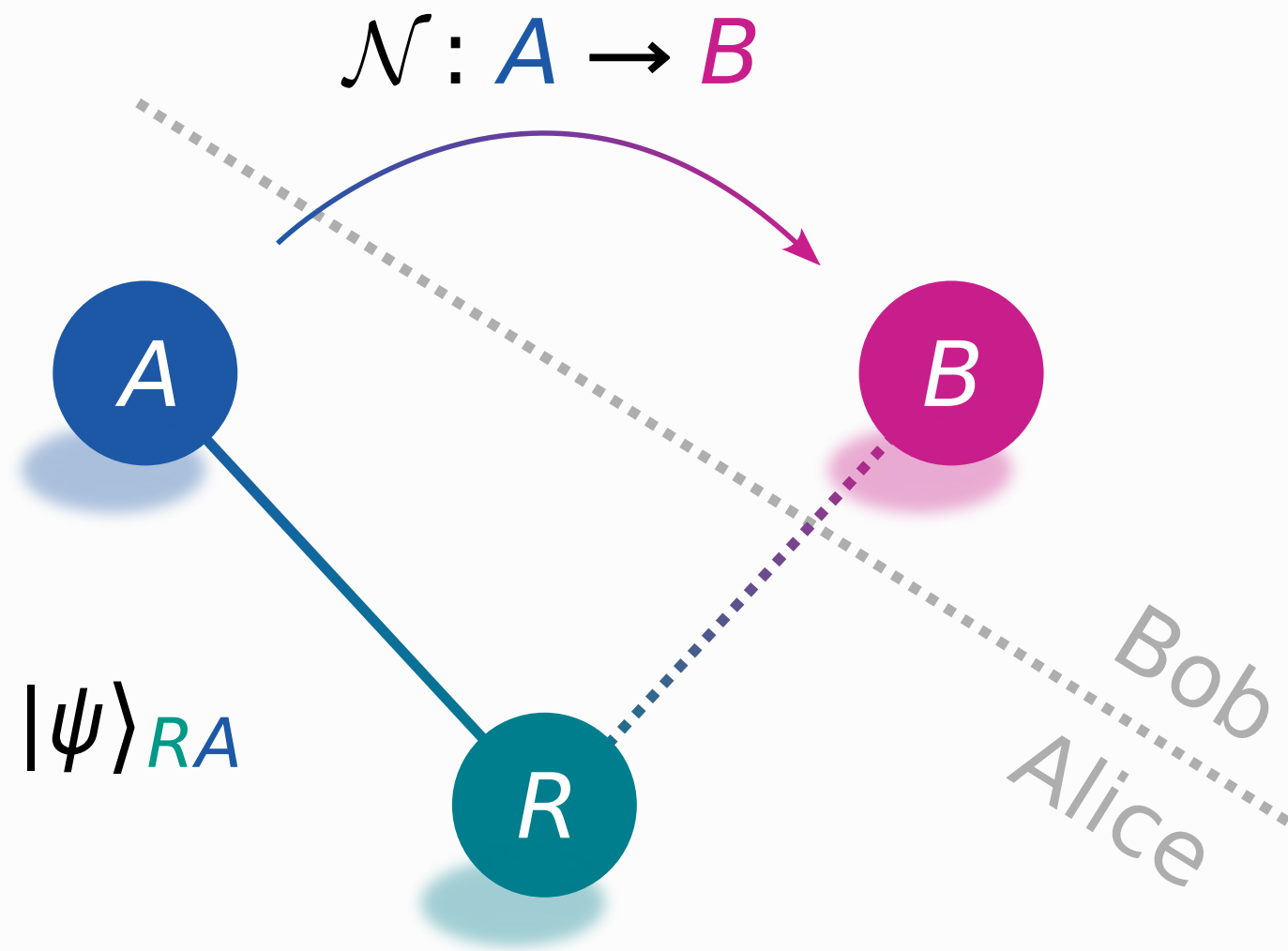
Coherent information $I_c(\mathcal{N}, \psi) = S(B)_\sigma - S(RB)_\sigma$



Interpretation of $I_c(\mathcal{N}, \psi)$:

Correlations established between **Alice** and **Bob** that survived the noisy channel action.

Quantifying quantum correlations



Channel coherent information

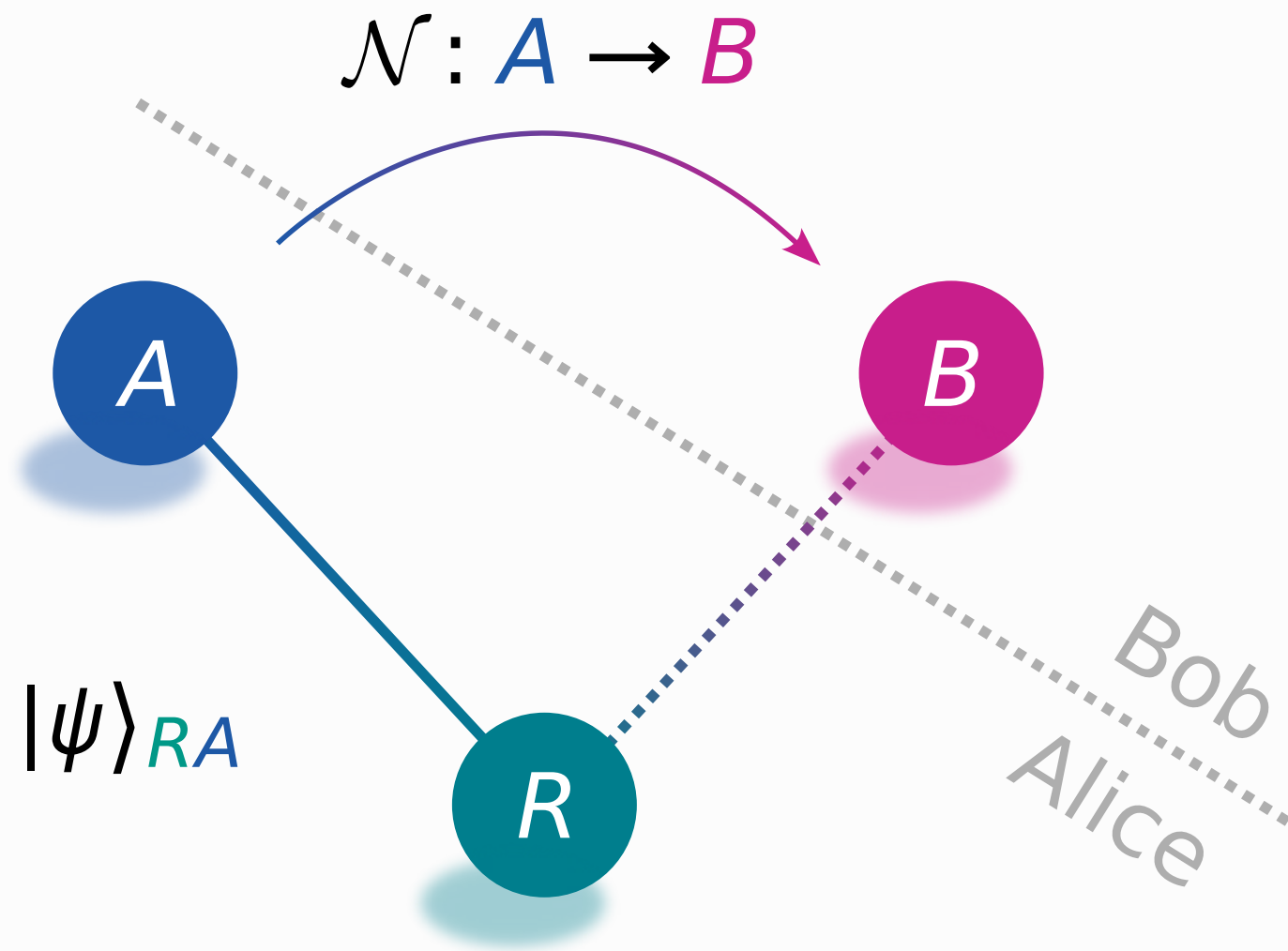
Choose best initial state ψ_{RA}
between **reference** and **channel** qubit:

$$I_c(\mathcal{N}) = \max_{\psi} I_c(\mathcal{N}, \psi)$$

Quantifying quantum correlations



Channel coherent information $I_c(\mathcal{N}) = \max_{\psi} I_c(\mathcal{N}, \psi)$

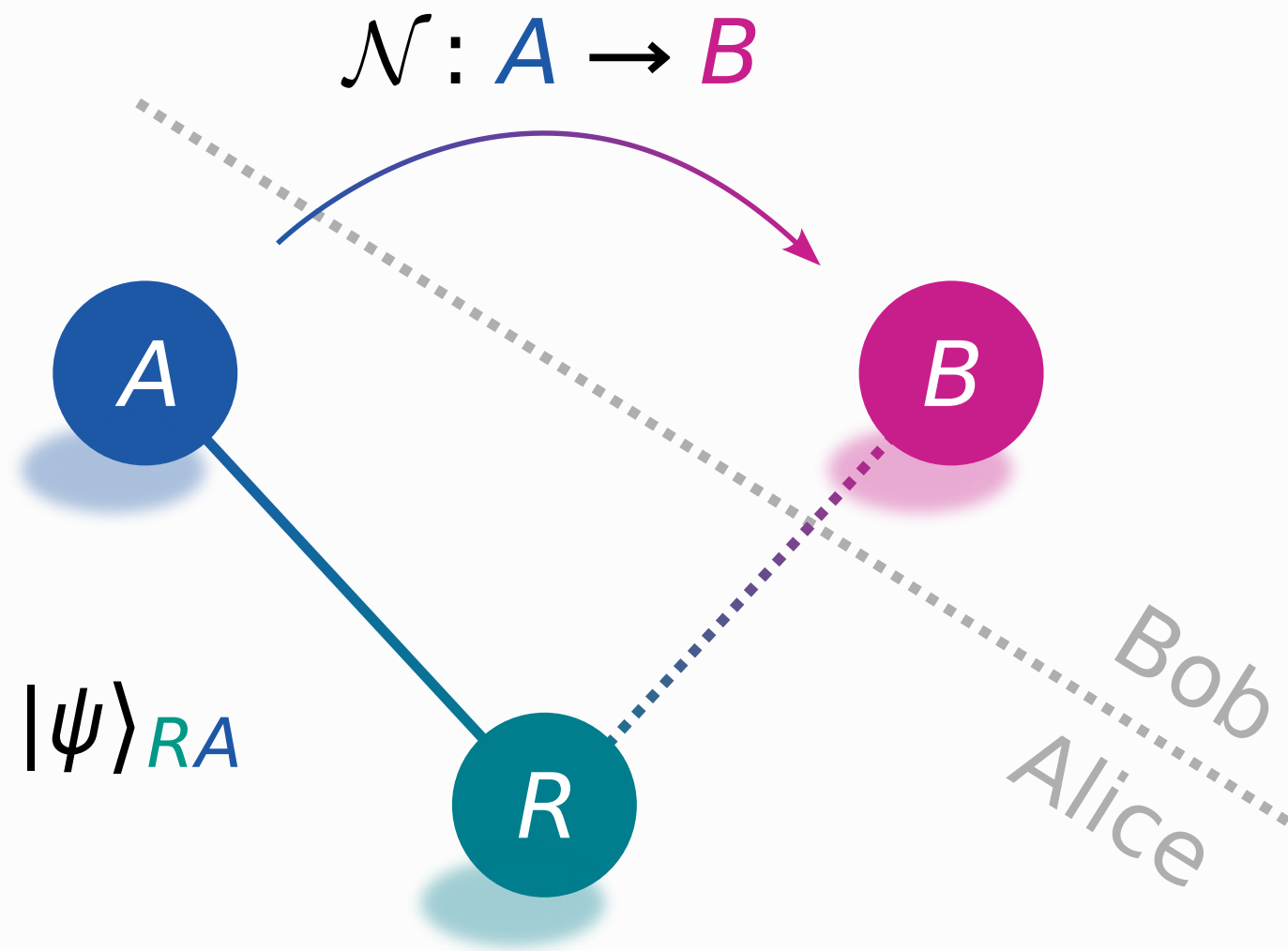


Quantum capacity theorem:
 $I_c(\mathcal{N})$ many ebits per channel use
can be recovered with fidelity ≈ 1
in the limit $n \rightarrow \infty$.

Quantifying quantum correlations



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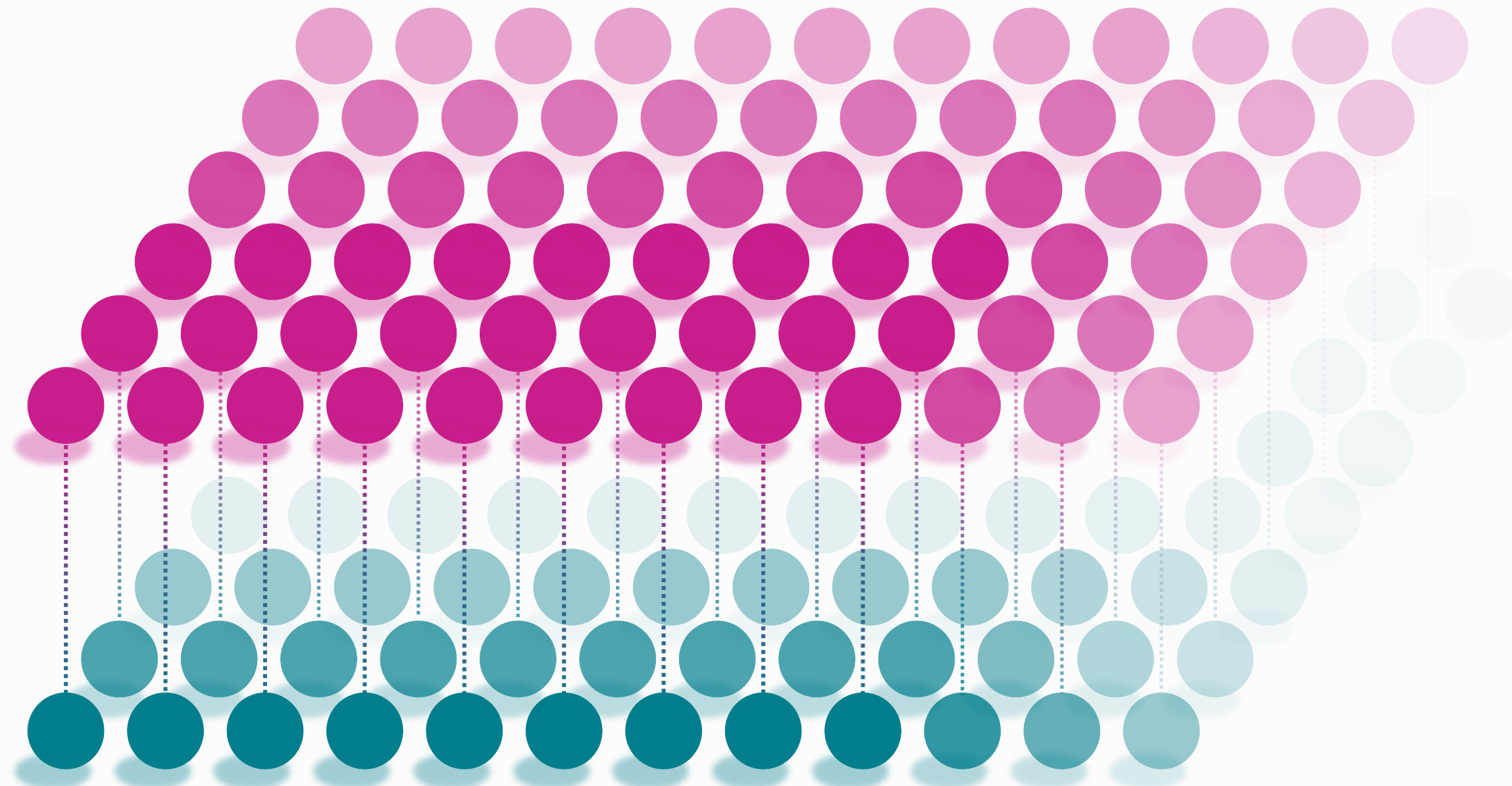
Clearest proof:

Hayden et al., quant-ph/0702005

Superadditivity of coherent information



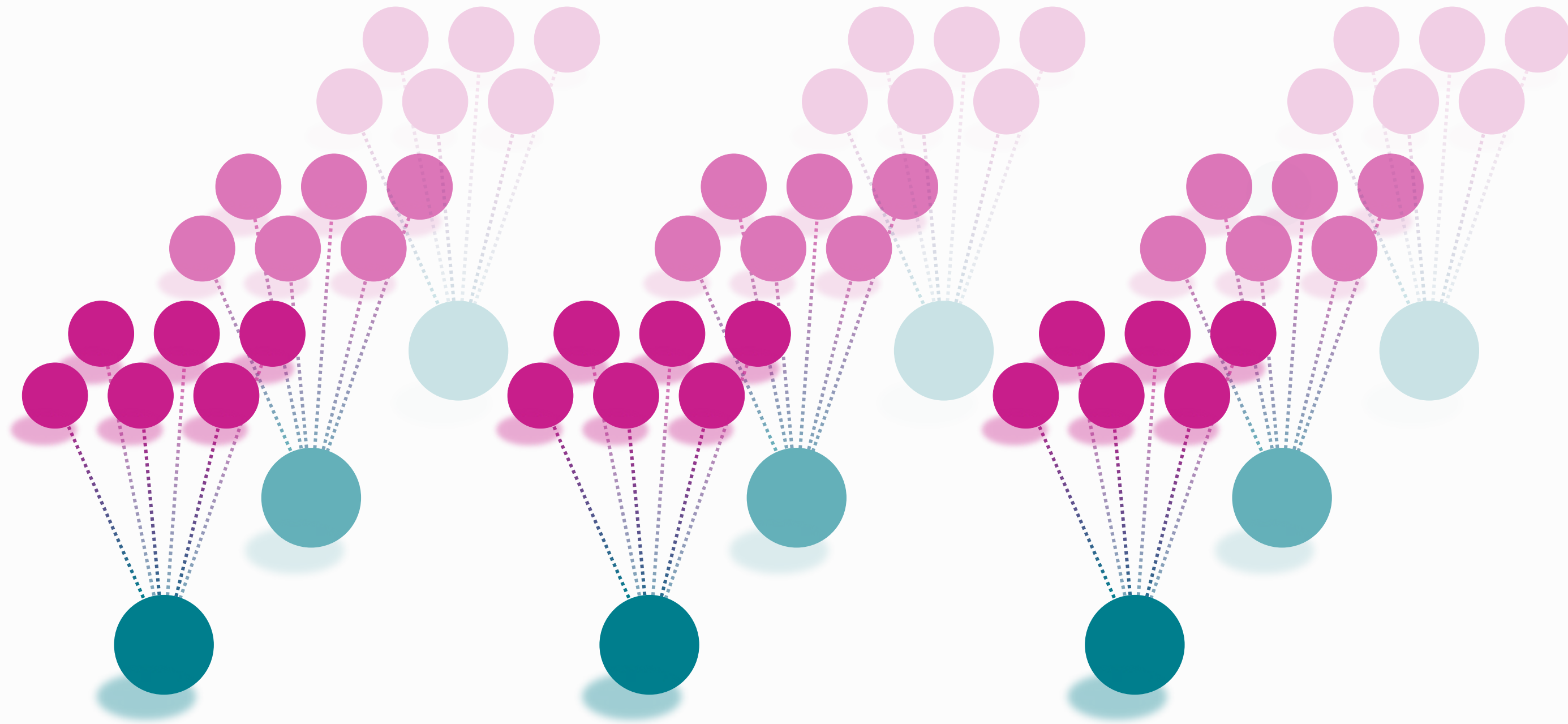
Recall: We aim to quantify surviving correlations to a reference system.



Superadditivity of coherent information



Idea: Use code blocks consisting of k entangled qubits.

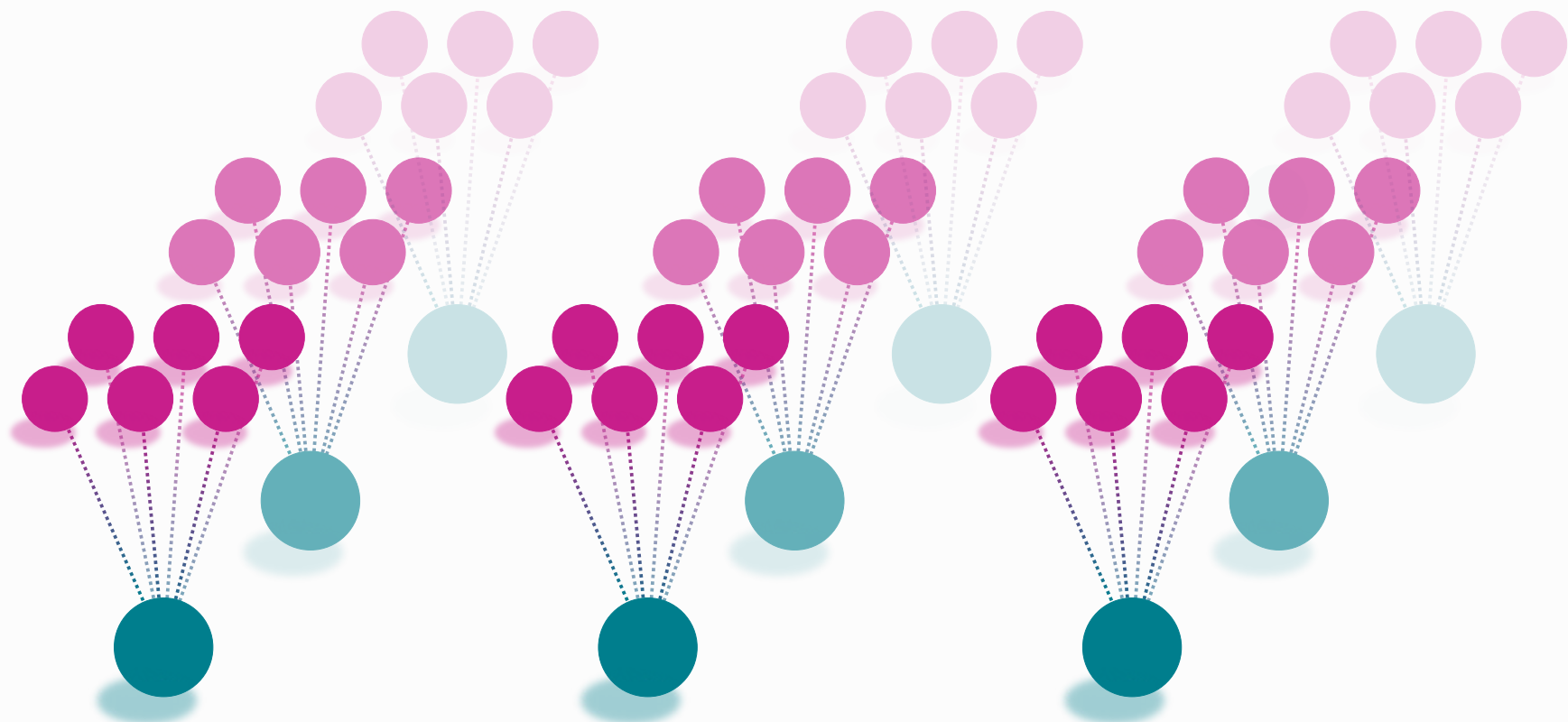


Superadditivity of coherent information

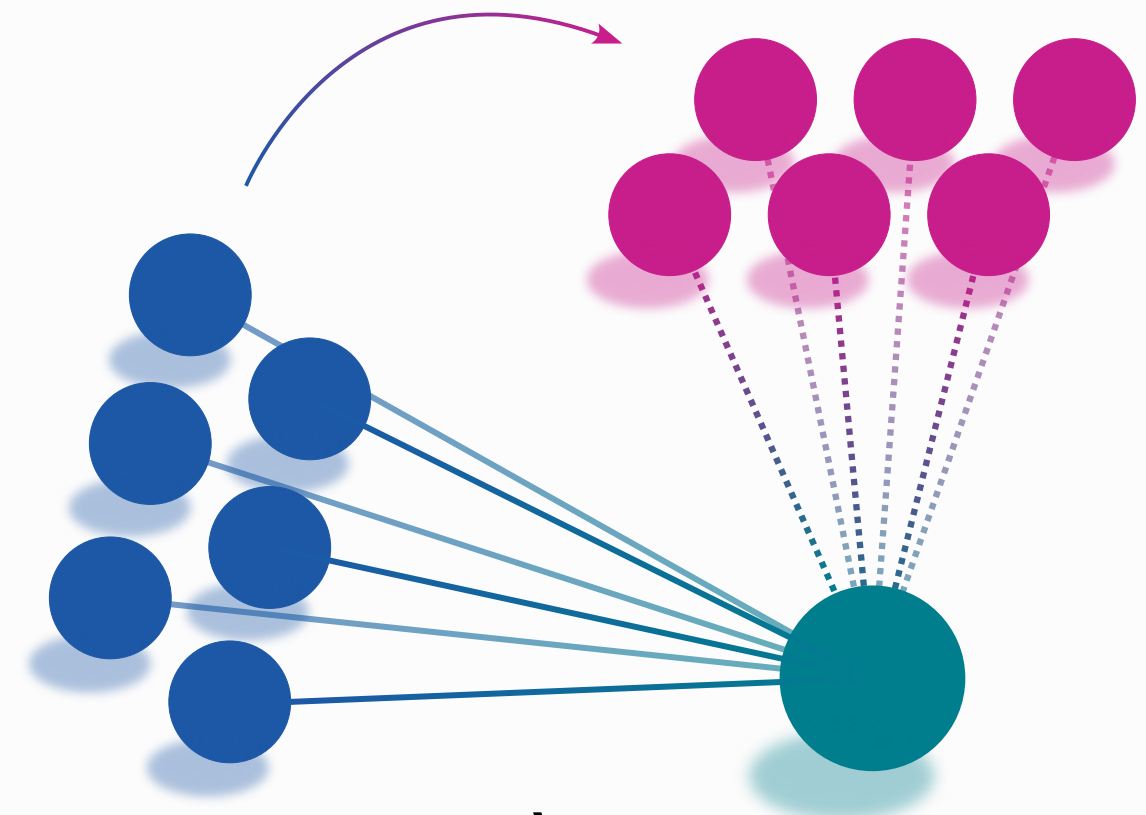


This creates blocks of k qubits in a state $|\psi_k\rangle_{RA^k}$

with coherent information $\frac{1}{k}I_c(\mathcal{N}^{\otimes k}, \psi_k)$.



$$\mathcal{N}^{\otimes k} : A^k \rightarrow B^k$$

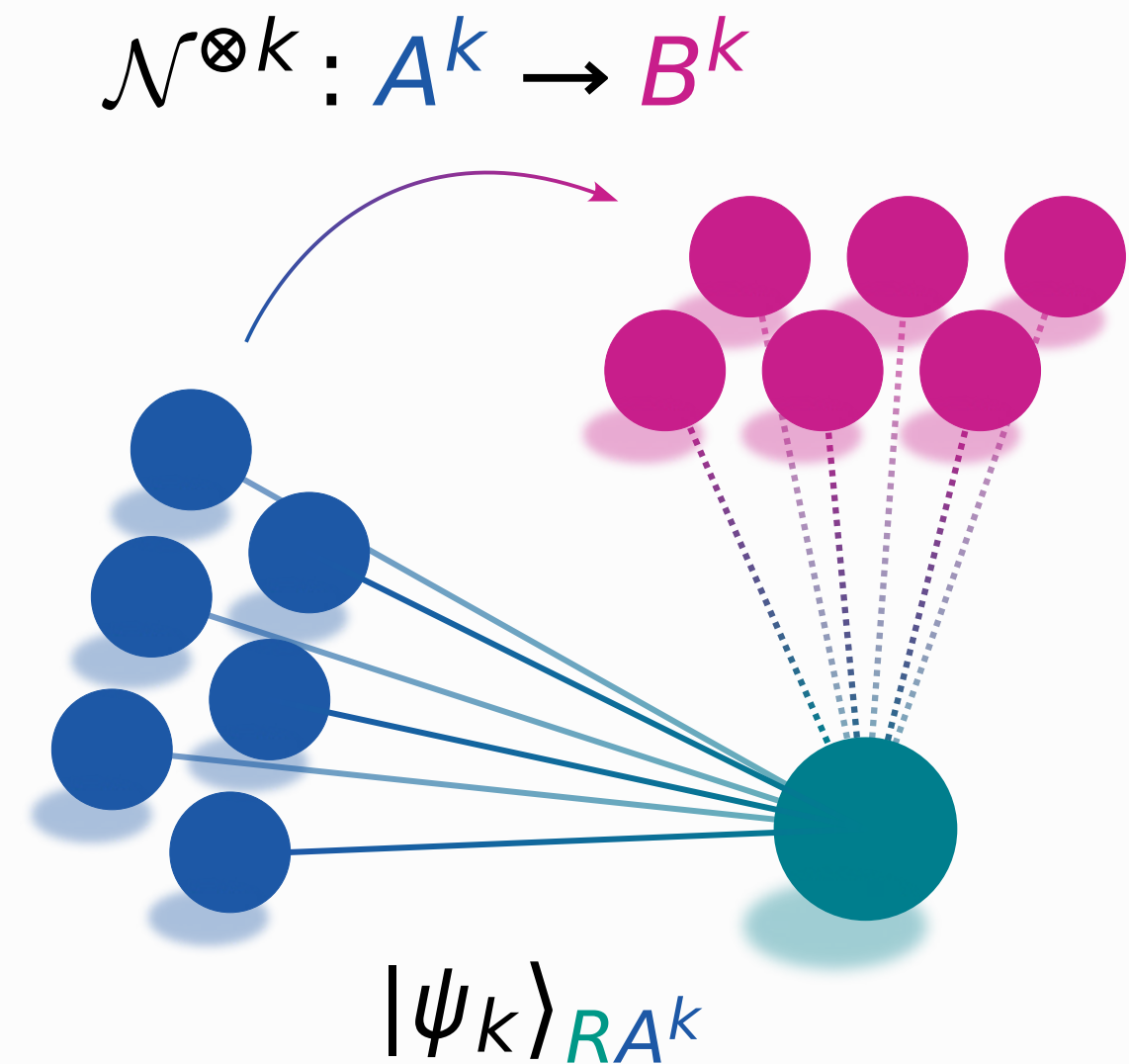


$$|\psi_k\rangle_{RA^k}$$

Superadditivity of coherent information



With $n = m \cdot k$ system qubits and taking $m \rightarrow \infty$, we can recover $\frac{1}{k} I_c(\mathcal{N}^{\otimes k})$ ebits per channel use with fidelity ≈ 1 .



Superadditivity of coherent information

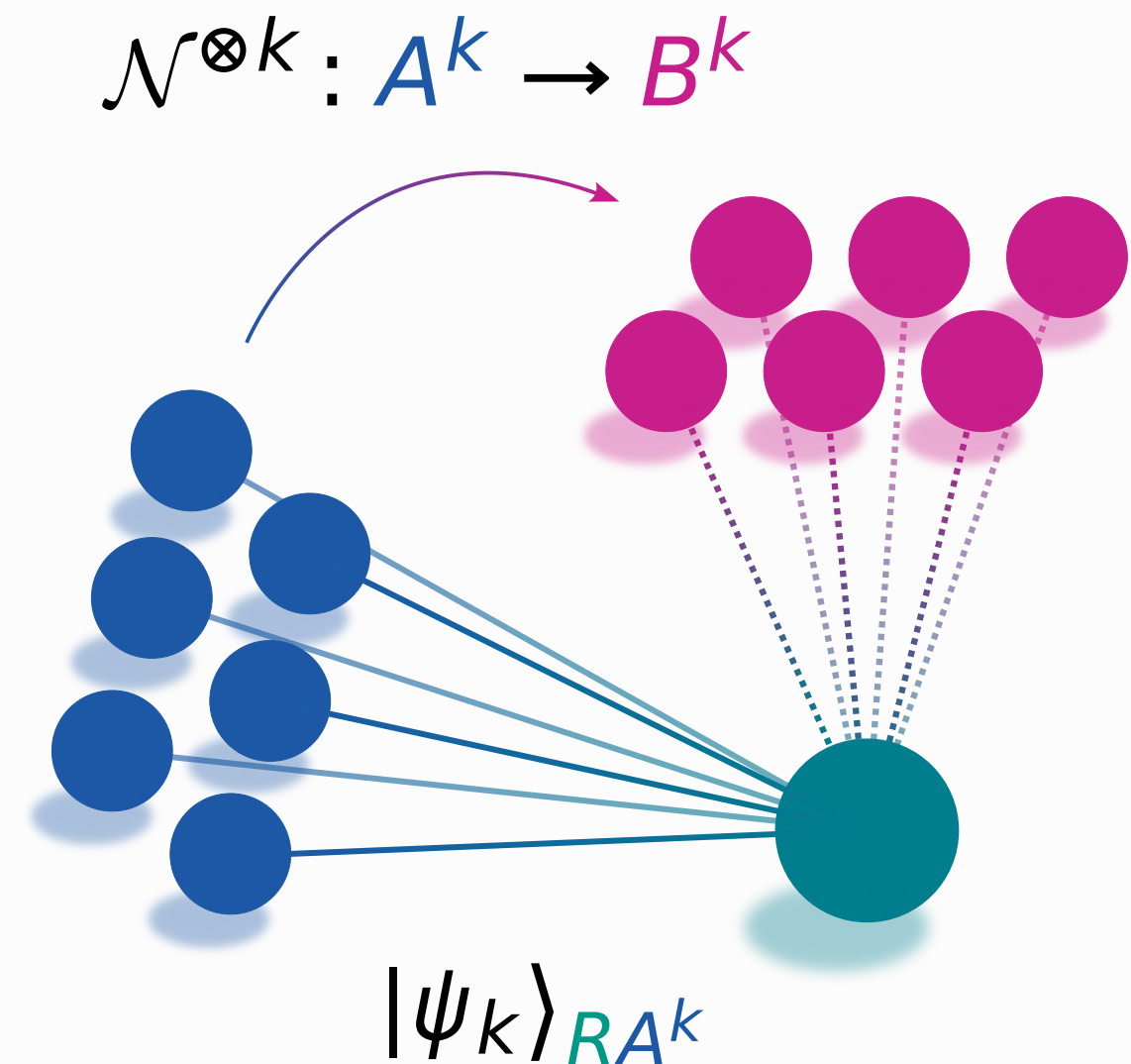


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Advantage:

Possible superadditivity of coherent information,

$$\frac{1}{k} I_c(\mathcal{N}^{\otimes k}) > I_c(\mathcal{N}).$$



Shor & Smolin '96, DiVincenzo et al. '98, Smith & Smolin '07,...

Quantum capacity



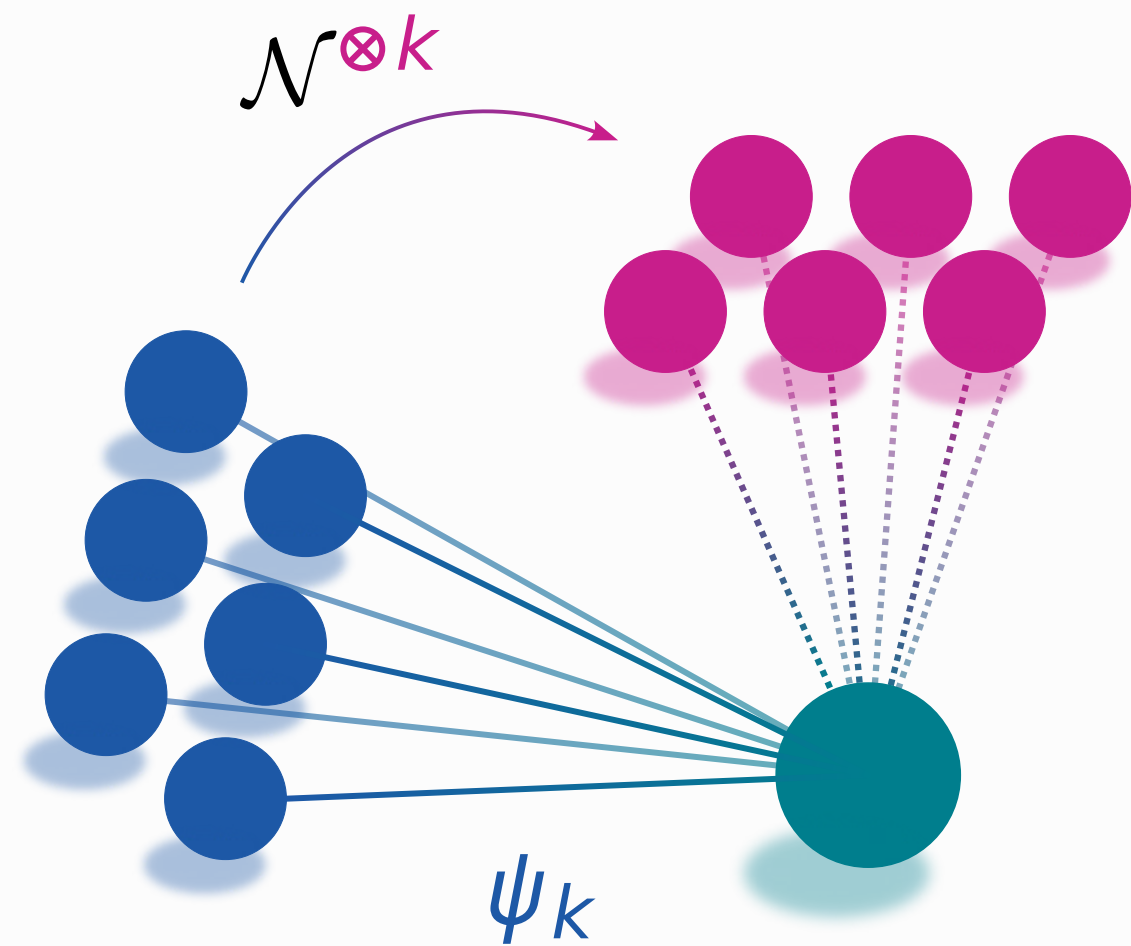
Quantum capacity $Q(\mathcal{N})$: Maximum rate at which ebits can be recovered.

Entropic formula:
$$Q(\mathcal{N}) = \sup_{k \in \mathbb{N}} \frac{1}{k} I_c(\mathcal{N}^{\otimes k})$$

Quantum capacity



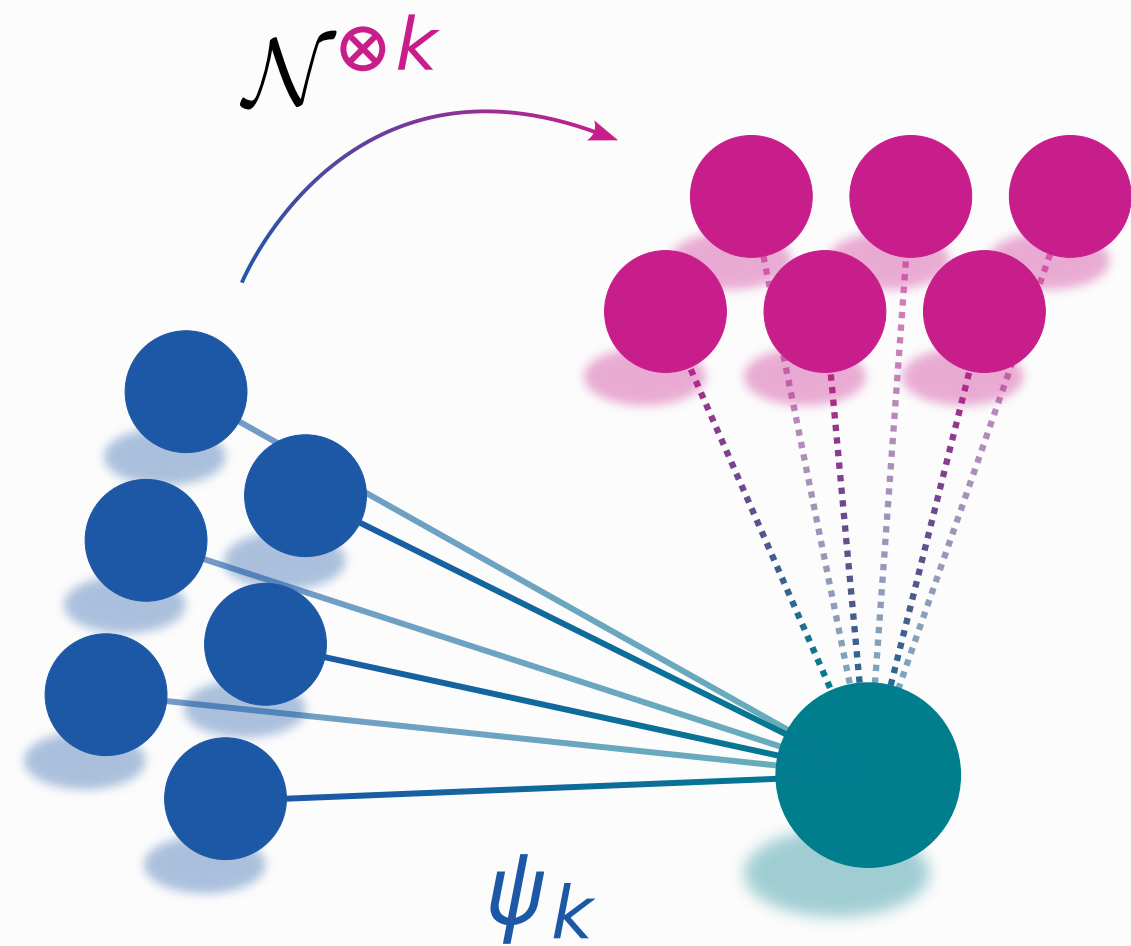
Entropic formula: $Q(\mathcal{N}) = \sup_{k \in \mathbb{N}} \max_{\psi_k} \frac{1}{k} I_c(\mathcal{N}^{\otimes k}, \psi_k)$



Quantum capacity



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Many problems:

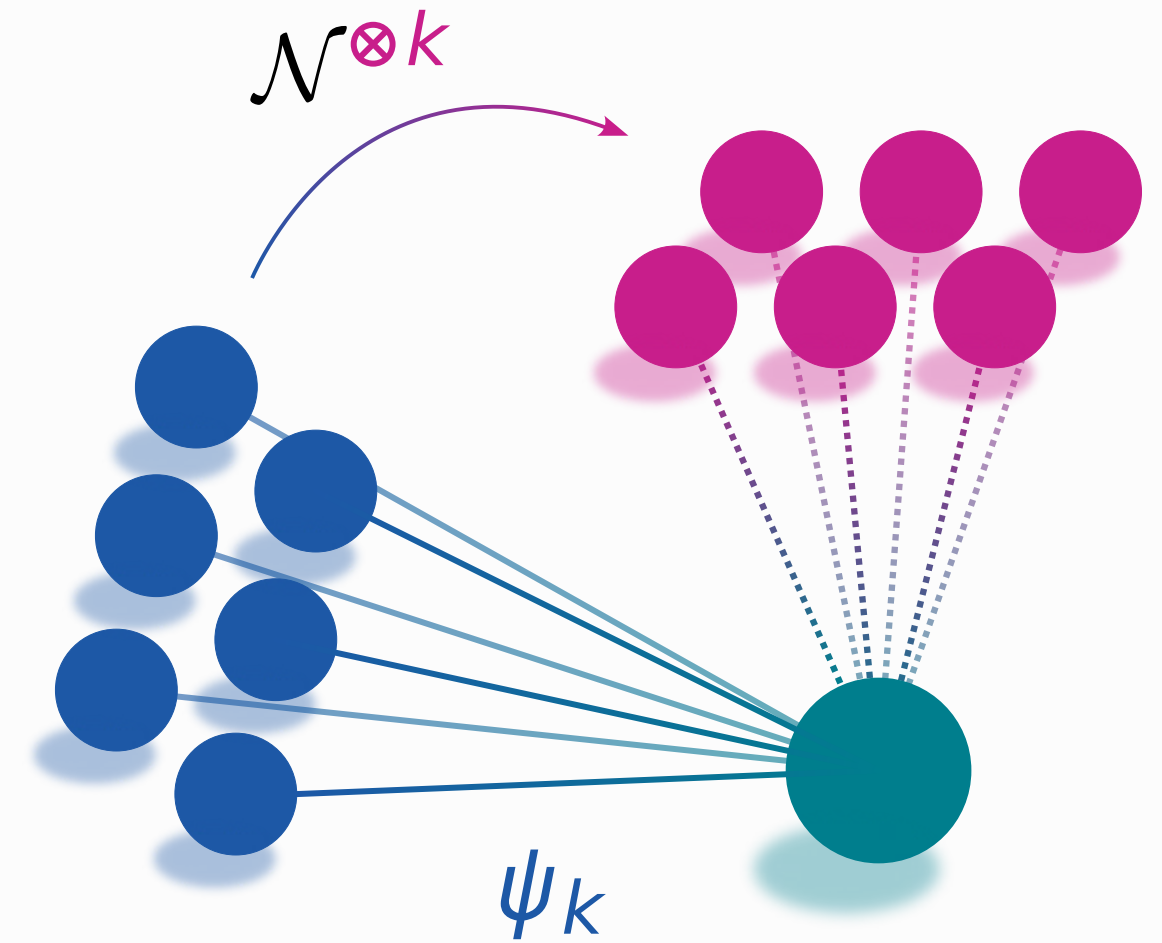
Unboundedness, non-convexity,
multipartite entanglement, ...

Lower bounds on quantum capacity



$$Q(\mathcal{N}) = \sup_{k \in \mathbb{N}} \max_{\psi_k} \frac{1}{k} I_c(\mathcal{N}^{\otimes k}, \psi_k)$$

This work: Focus on lower bounds



Lower bounds on quantum capacity

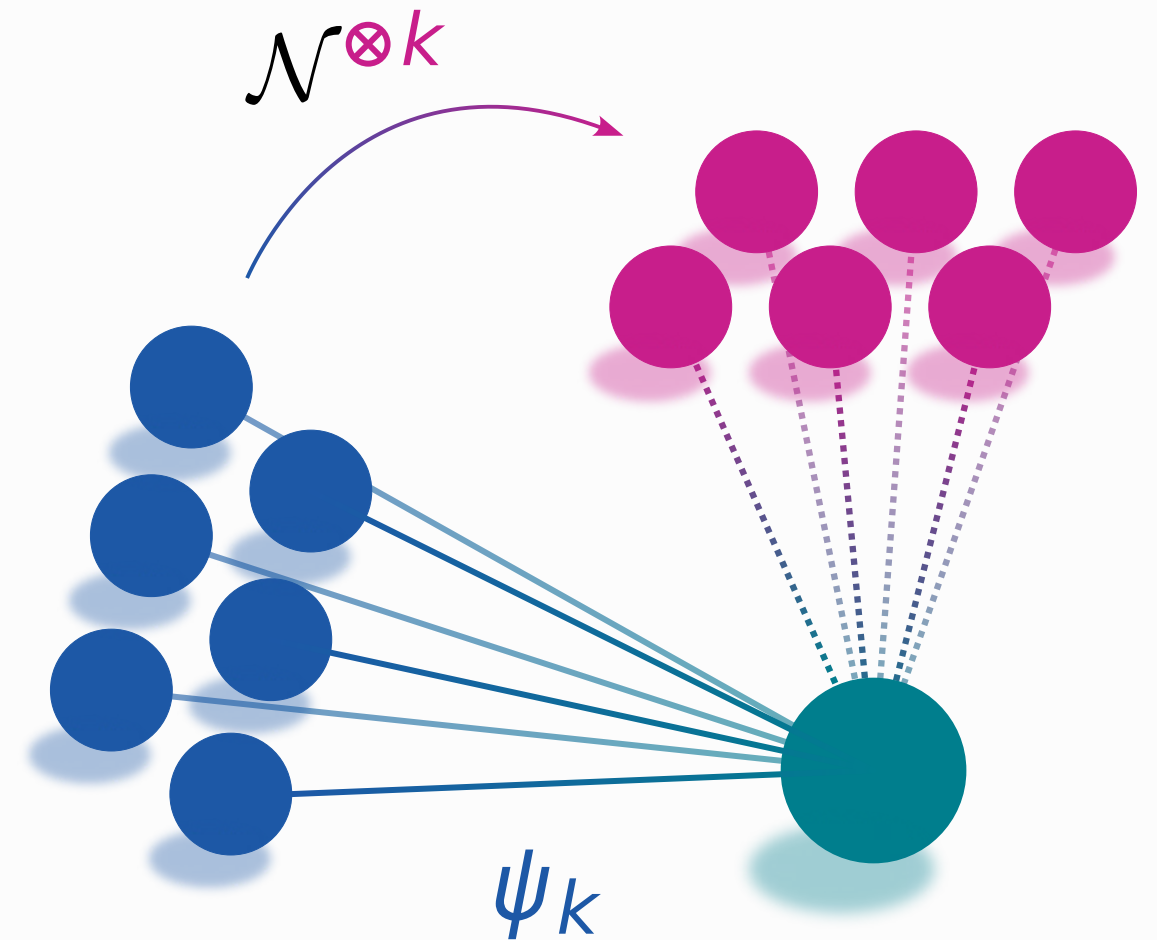


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This work: Focus on lower bounds

Fix $k = \#$ channel qubits and find ψ_k such that

$$Q(\mathcal{N}) \geq \frac{1}{k} I_c(\mathcal{N}^{\otimes k}, \psi_k) > I_c(\mathcal{N}).$$



Lower bounds on quantum capacity

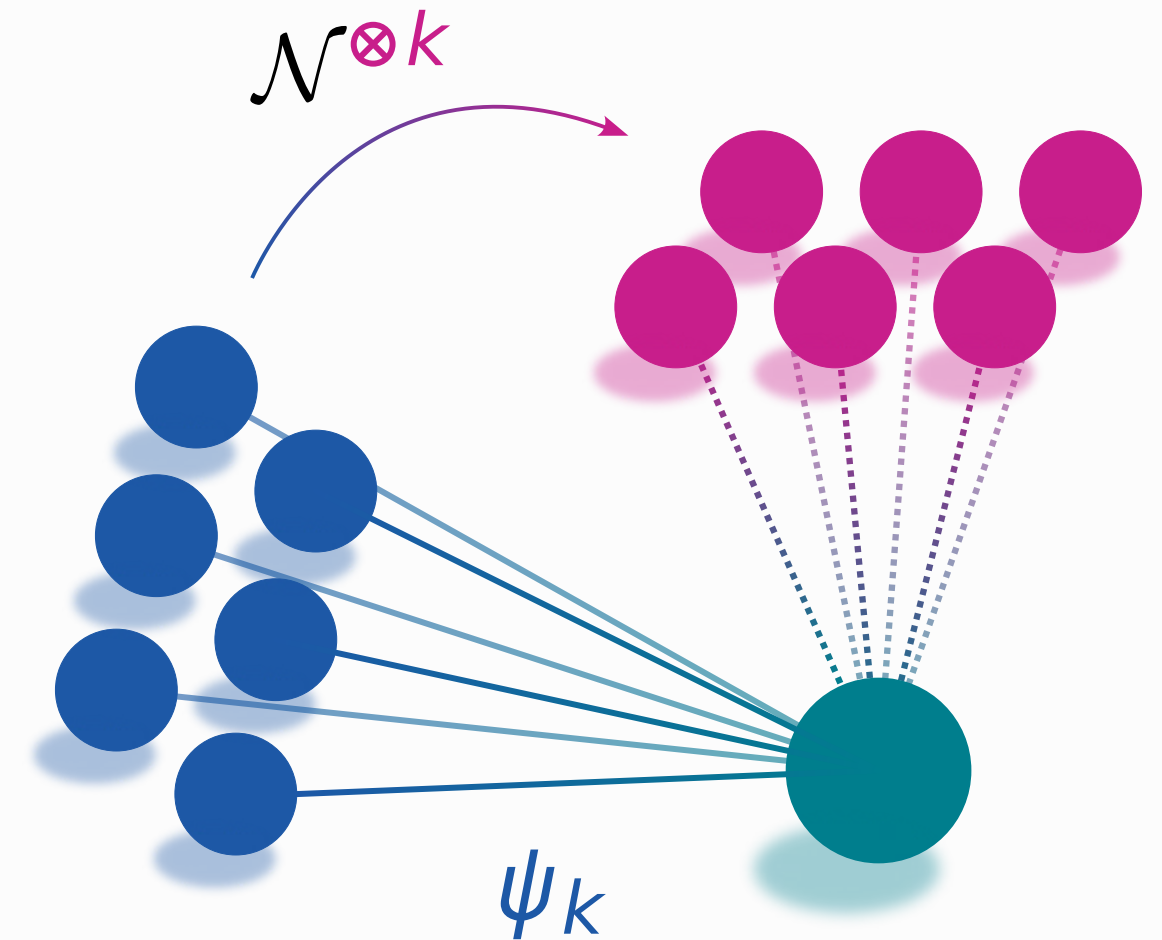


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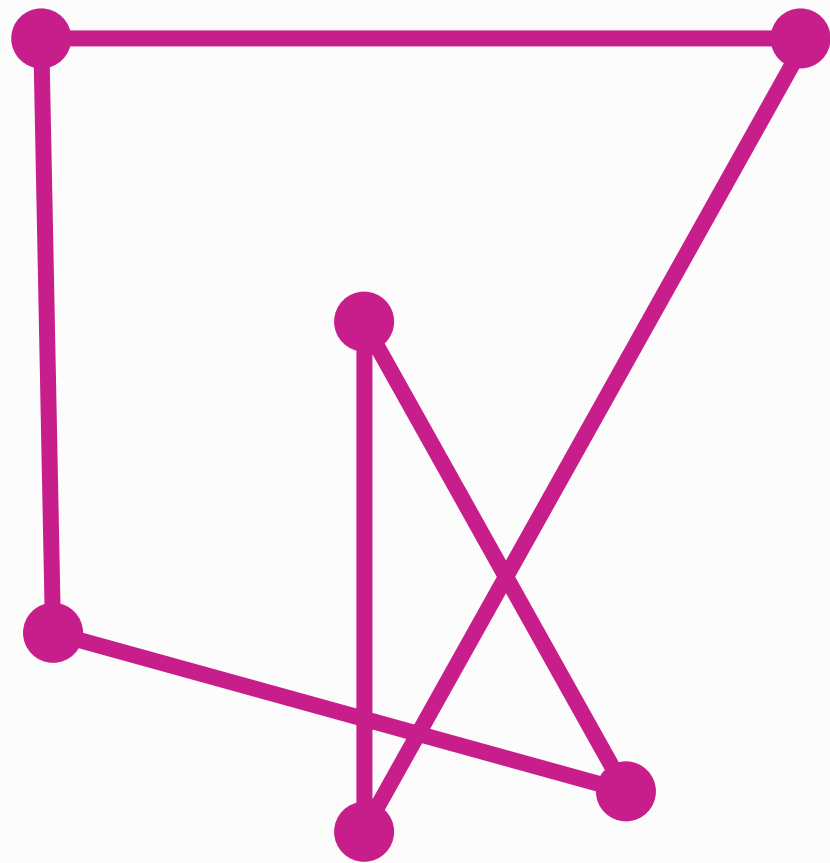
How to efficiently compute coherent information for larger k?

Symmetries in mathematical physics

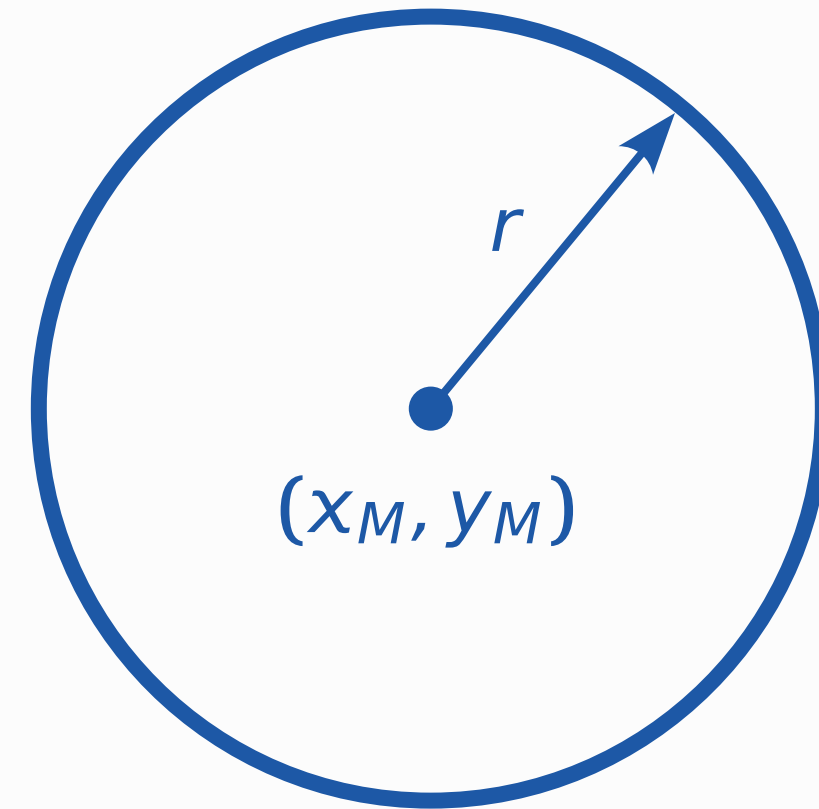


Symmetry eliminates free parameters:

"Do something and nothing happens."




no symmetries,
complicated description

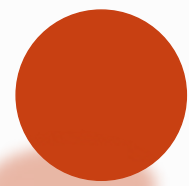
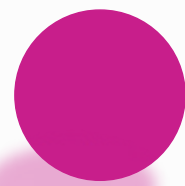
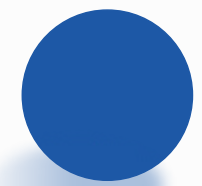


rotational symmetry,
described by three parameters

Symmetries in quantum information



Permuting k qudits 



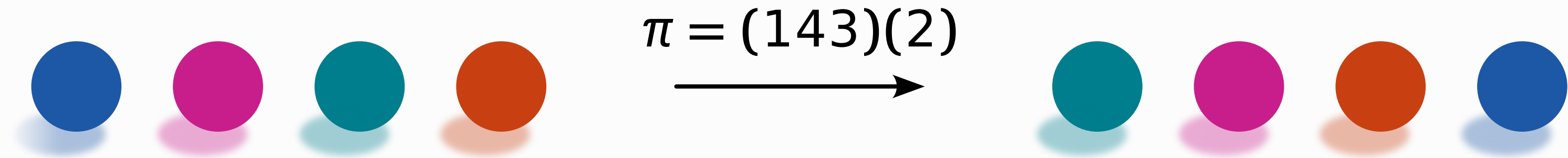
$$\pi = (143)(2)$$



Symmetries in quantum information



Permuting k qudits 



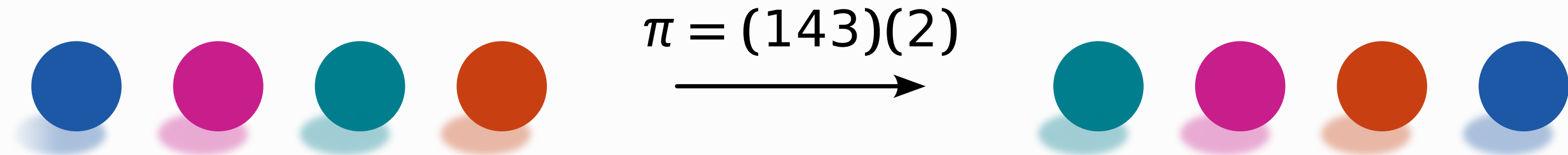
This action commutes with an i.i.d. channel $\mathcal{N}^{\otimes k}$:

$$\mathcal{N}^{\otimes 4}(\text{blue circle, pink circle, teal circle, orange circle}) = \mathcal{N}(\text{blue circle})\mathcal{N}(\text{pink circle})\mathcal{N}(\text{teal circle})\mathcal{N}(\text{orange circle})$$

Symmetries in quantum information



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$$\mathcal{N}(\text{teal})\mathcal{N}(\text{pink})\mathcal{N}(\text{orange})\mathcal{N}(\text{blue})$$

$\pi = (143)(2)$

Decomposing representations



Permuting qudits is a **representation** of the symmetric group S_k on the Hilbert space $(\mathbb{C}^d)^{\otimes k}$.

Decomposing representations



Permuting qudits is a **representation** of the symmetric group S_k on the Hilbert space $(\mathbb{C}^d)^{\otimes k}$.

We can decompose this representation into “building blocks” called **irreducible representations (irreps) S_λ** :

$$\begin{aligned}(\mathbb{C}^d)^{\otimes k} &\cong \bigoplus_{\lambda} S_{\lambda} \oplus \cdots \oplus S_{\lambda} \\ &\cong \bigoplus_{\lambda} S_{\lambda} \otimes M_{\lambda}\end{aligned}$$

λ labels inequivalent **irreps S_λ** , and M_λ is a **multiplicity space**.

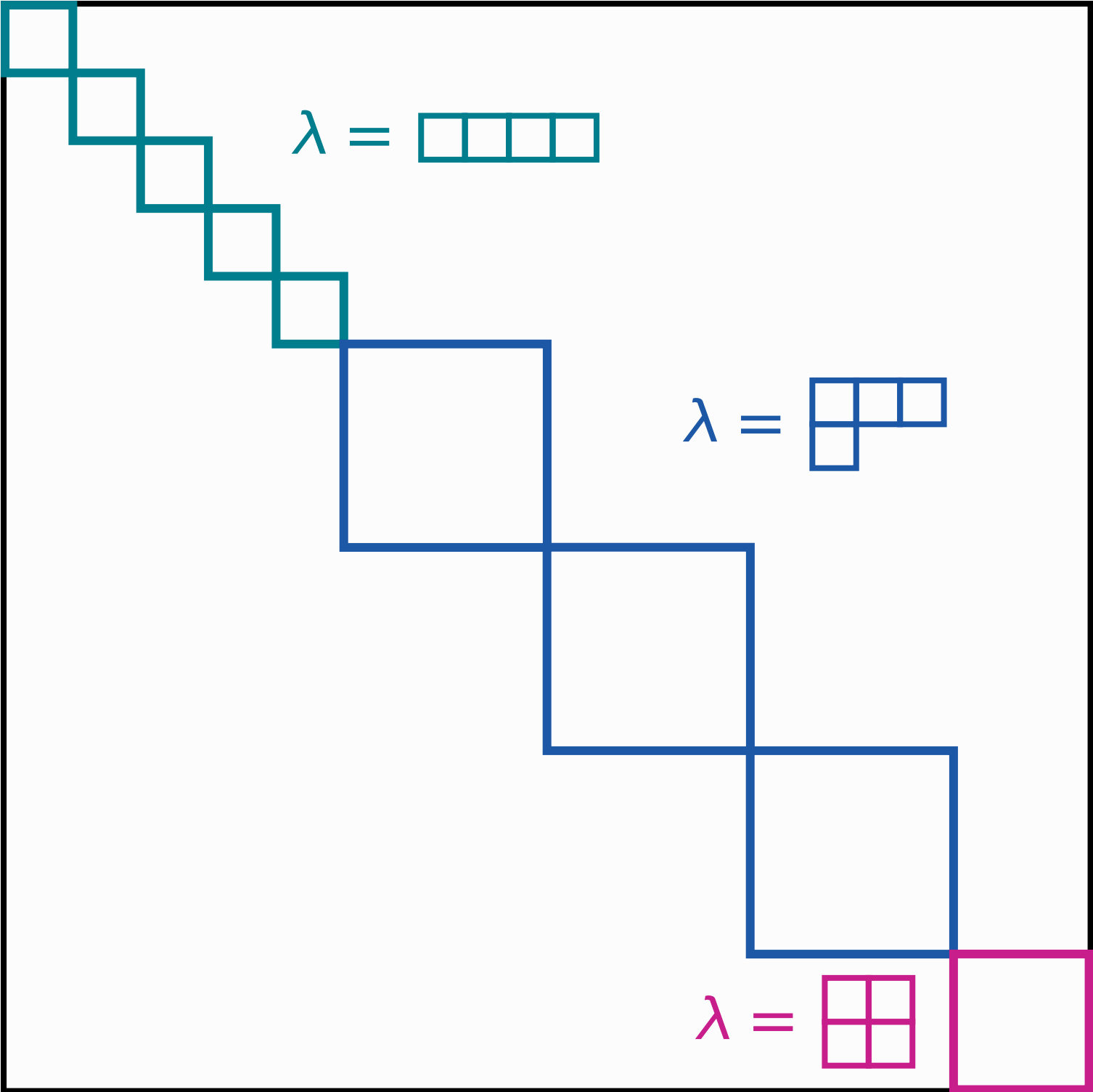
Decomposing representations



$$(\mathbb{C}^d)^{\otimes k} \cong \bigoplus_{\lambda} S_{\lambda} \otimes M_{\lambda}$$

λ			
$\dim S_{\lambda}$	1	3	2
$\dim M_{\lambda}$	5	3	1

Example: $d = 2, k = 4$



Decomposing representations



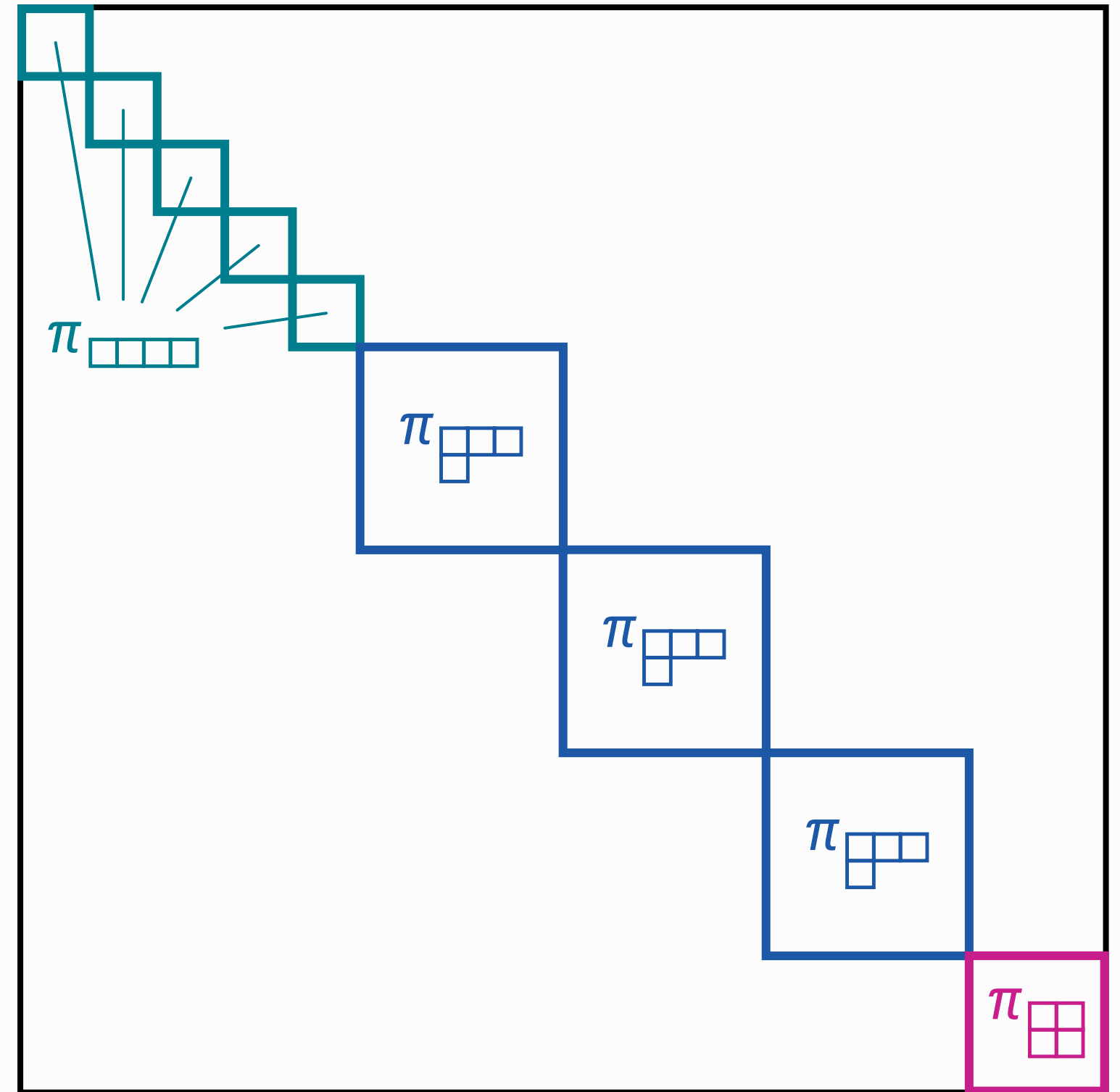
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Permutations only act on the S_{λ} :

$$\pi \cong \bigoplus_{\lambda} \pi_{\lambda} \otimes \mathbb{1}_{M_{\lambda}}$$

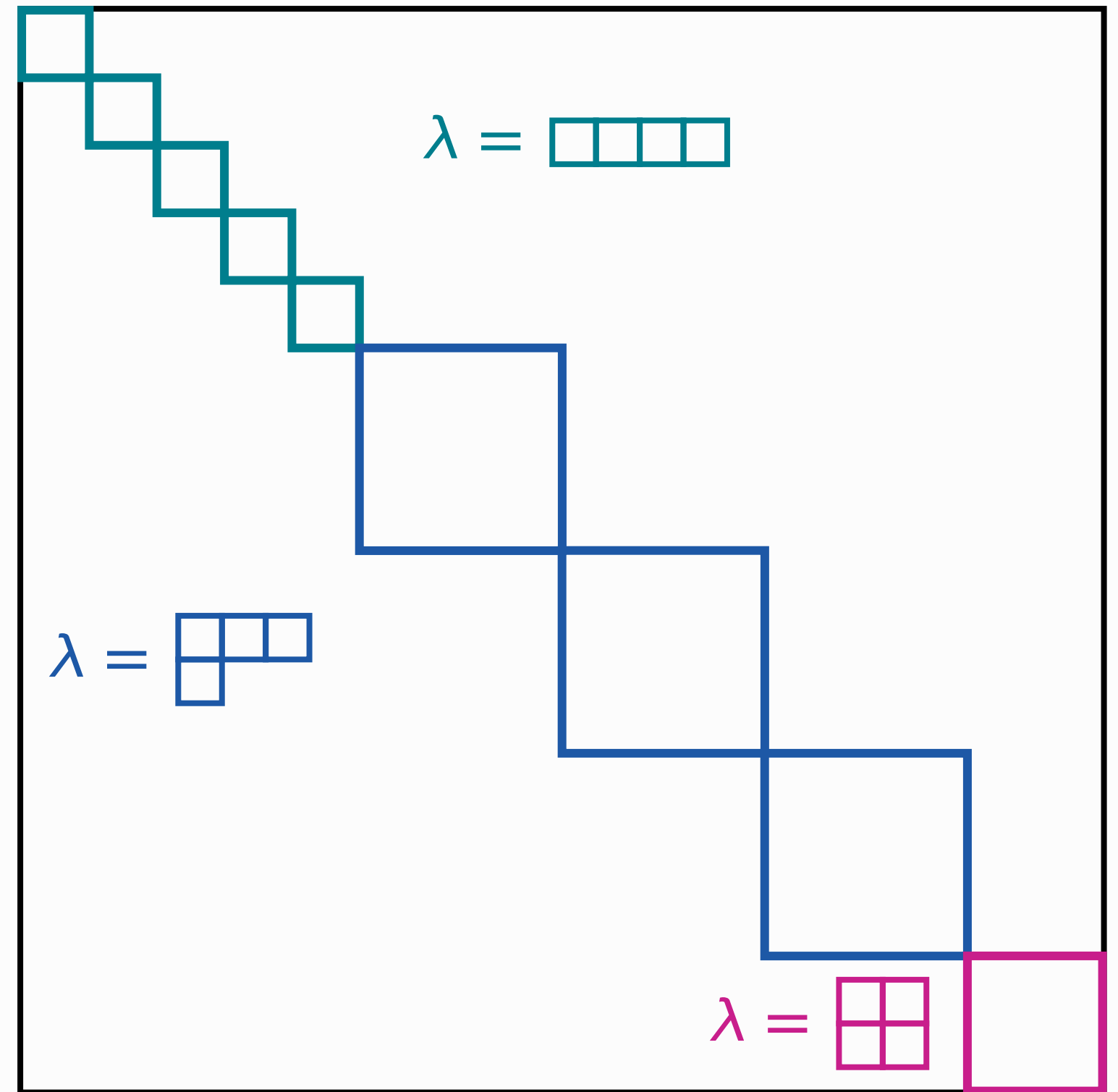


Invariant states



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Invariant states

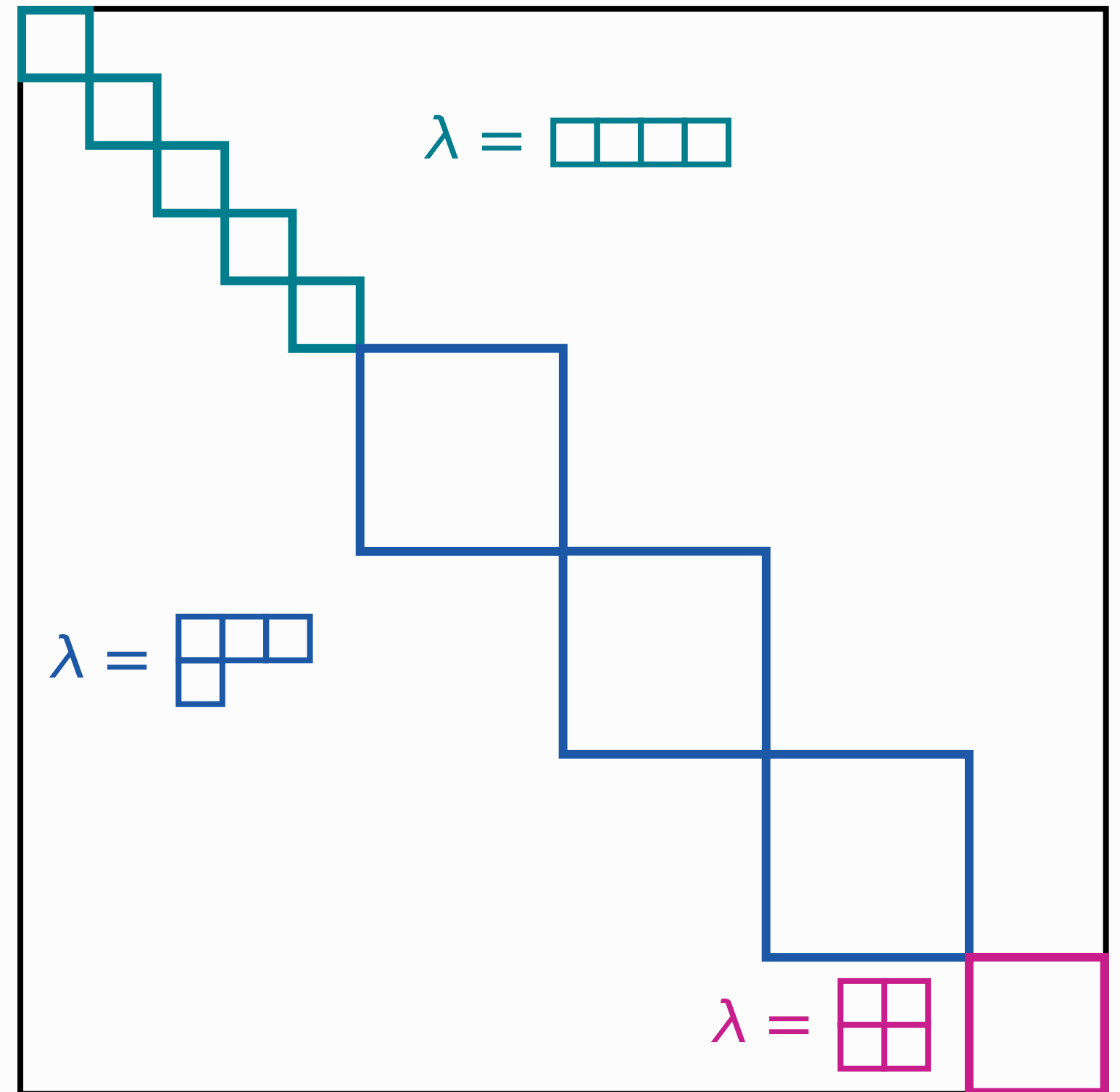


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$$[\rho, \pi] = 0 \text{ for all } \pi \in S_k,$$



Invariant states



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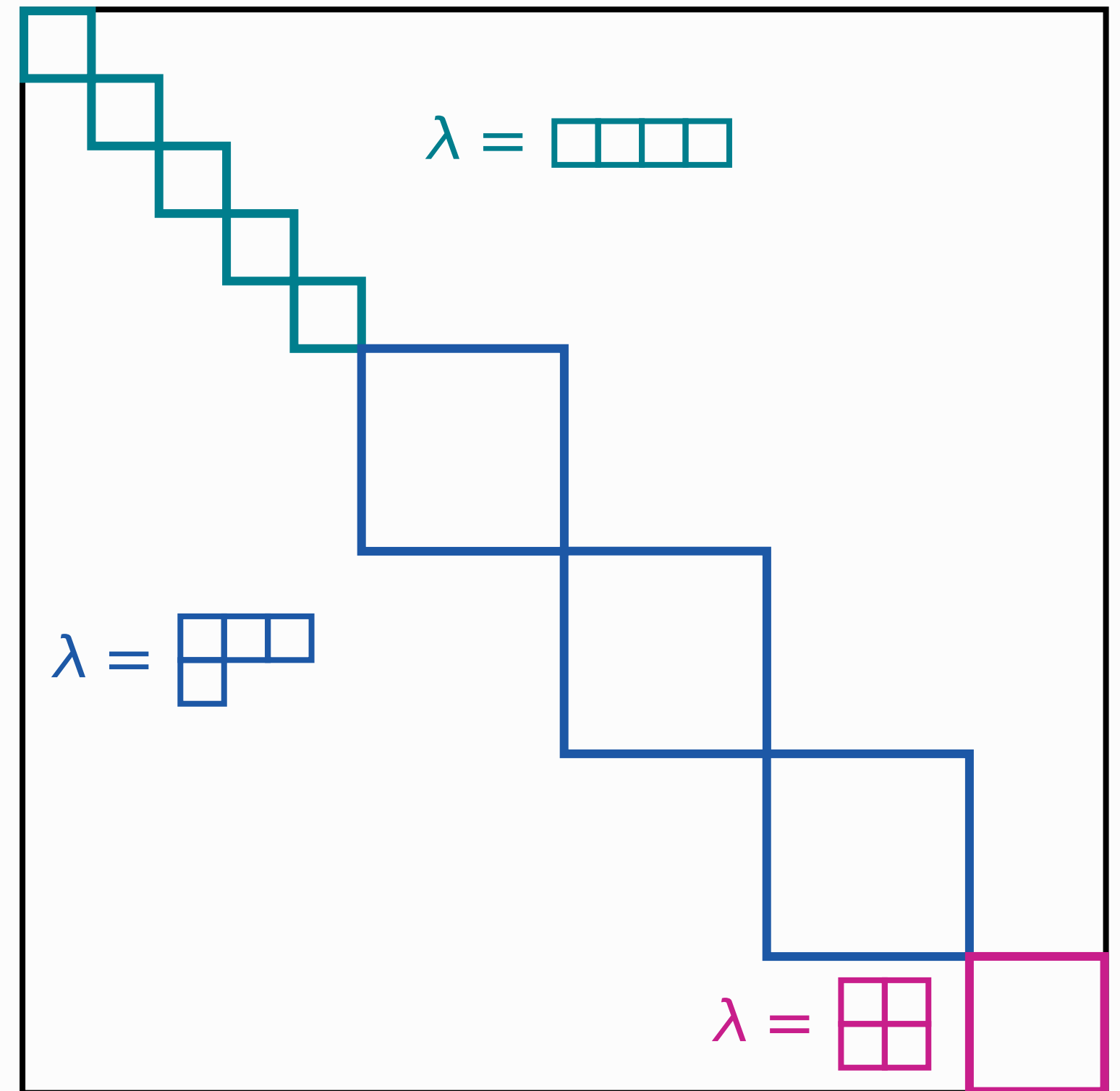
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Invariant states



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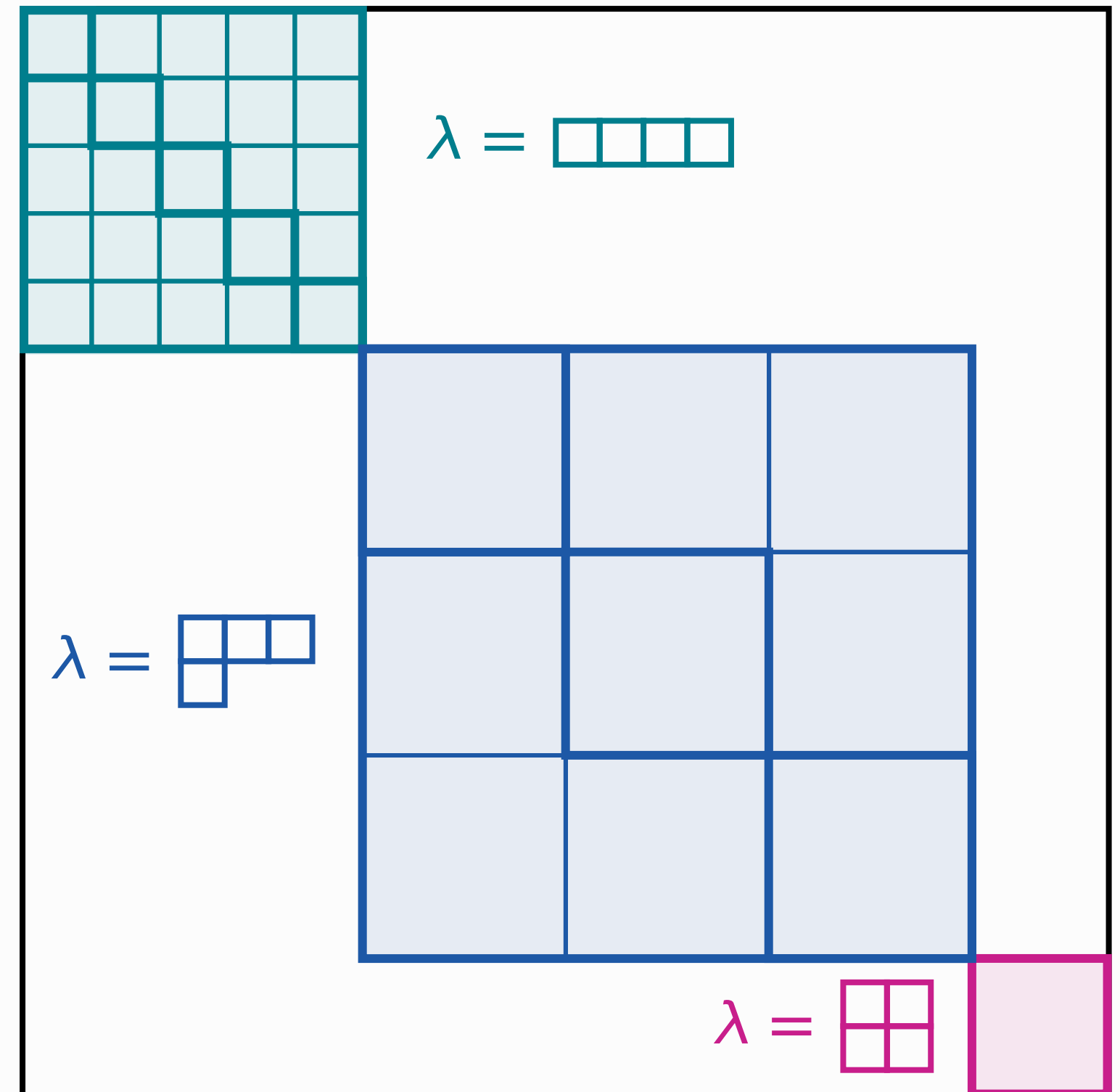
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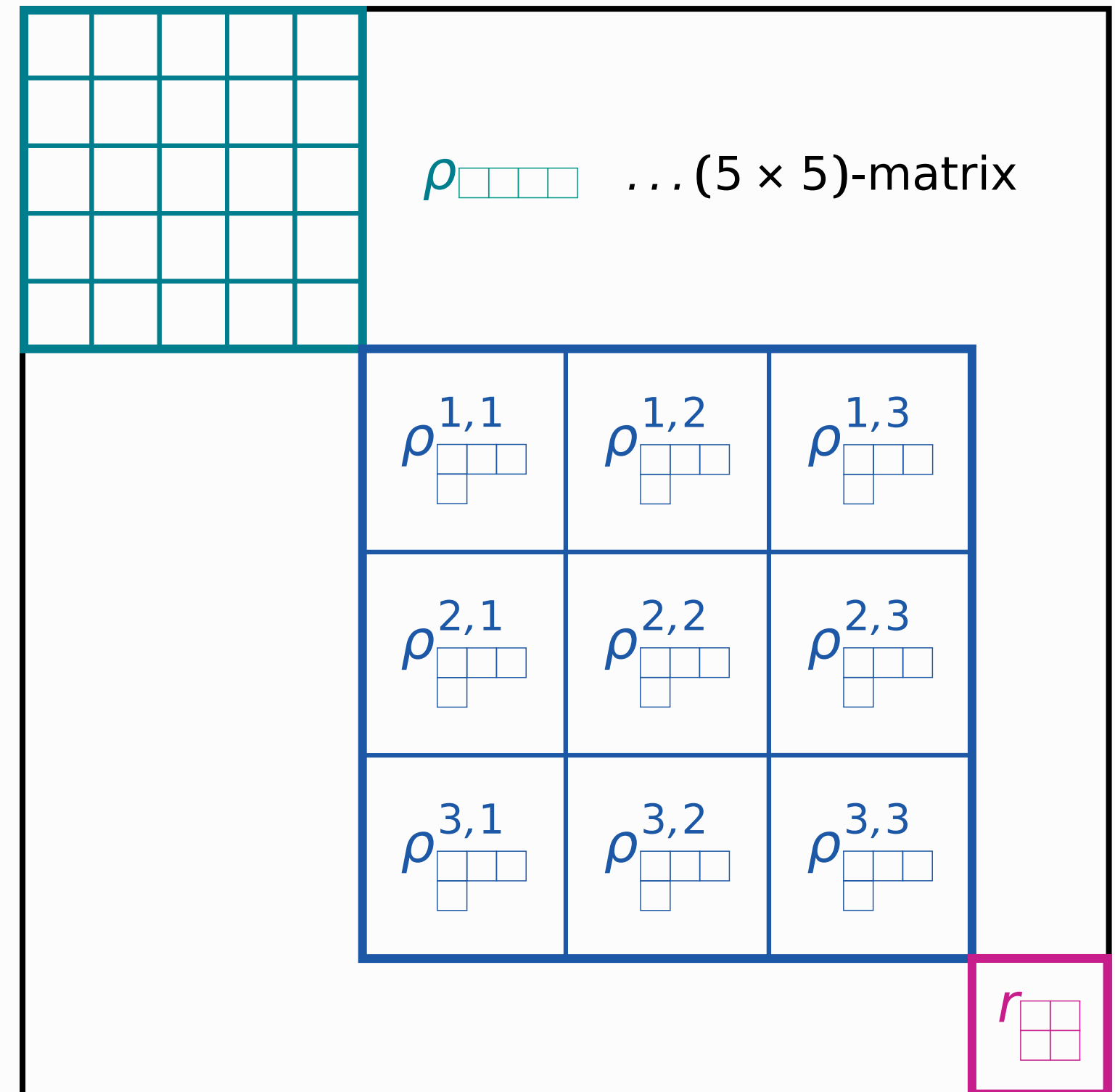
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Invariant states



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How can we determine the effective states ρ_{λ} ?



$\rho \cong \bigoplus_{\lambda} \mathbb{1}_{S_{\lambda}} \otimes \rho_{\lambda}$. How can we determine the effective states ρ_{λ} ?

Schur-Weyl duality

The multiplicity spaces M_{λ} in the decomposition

$$(\mathbb{C}^d)^{\otimes k} \cong \bigoplus_{\lambda} S_{\lambda} \otimes M_{\lambda}$$

are actually the irreps of the representation $GL(d) \ni g \mapsto g^{\otimes k}$ of the general linear group $GL(d)$.

Invariant states



Recall:

$$(\mathbb{C}^d)^{\otimes k} \cong \bigoplus_{\lambda} S_{\lambda} \otimes M_{\lambda}$$

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Invariant states



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Schur-Weyl duality: For any $X \in \mathcal{L}(\mathbb{C}^d)$,

$$X^{\otimes k} \cong \bigoplus_{\lambda} \mathbb{1}_{S_{\lambda}} \otimes X_{\lambda},$$

and there are efficient constructions of the X_{λ} .

Invariant states



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This lets us easily describe states of the form $\rho_{(k)} = \sum_i x_i \rho_i^{\otimes k}$.

Invariant states



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Well-known example: Repetition code

$$\rho_{(k)} = \frac{1}{2} (|0\rangle\langle 0|^{\otimes k} + |1\rangle\langle 1|^{\otimes k})$$

with purification $|\psi_k\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |0\rangle^{\otimes k} + |1\rangle \otimes |1\rangle^{\otimes k})$.

Invariant states



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This code gives **superadditive coherent information** for Pauli channels

$$\rho \longmapsto (1-p)\rho + p[q_1 X\rho X + q_2 Y\rho Y + q_3 Z\rho Z]$$

Main theoretical result



For states of the form $\rho_{(k)} = \sum_i x_i \rho_i^{\otimes k} \cong \bigoplus_{\lambda} \mathbb{1}_{S_{\lambda}} \otimes \sum_i x_i \rho_i^{(\lambda)}$,

the coherent information $I_c(\mathcal{N}^{\otimes k}, \rho_{(k)})$ assumes the form

$$I_c(\mathcal{N}^{\otimes k}, \rho_{(k)}) = \sum_{\lambda} f\left(\mathcal{N}, \sum_i x_i \rho_i^{(\lambda)}\right).$$

Main theoretical result



For states of the form $\rho_{(k)} = \sum_i x_i \rho_i^{\otimes k} \cong \bigoplus_{\lambda} \mathbb{1}_{S_{\lambda}} \otimes \sum_i x_i \rho_i^{(\lambda)}$,

the coherent information $I_c(\mathcal{N}^{\otimes k}, \rho_{(k)})$ assumes the form

$$I_c(\mathcal{N}^{\otimes k}, \rho_{(k)}) = \sum_{\lambda} f\left(\mathcal{N}, \sum_i x_i \rho_i^{(\lambda)}\right).$$

Since $|\{\lambda\}|, \dim M_{\lambda} = O(\text{poly}(n))$, this formula is **efficiently computable**

for up to $n \approx 100$ for qubit channels!

Goal: Find lower bounds on the quantum capacity of channels such as

$$\mathcal{N}_p: \sigma \longmapsto (1-p)\sigma + p[q_1 X\sigma X + q_2 Y\sigma Y + q_3 Z\sigma Z]$$

Numerical studies

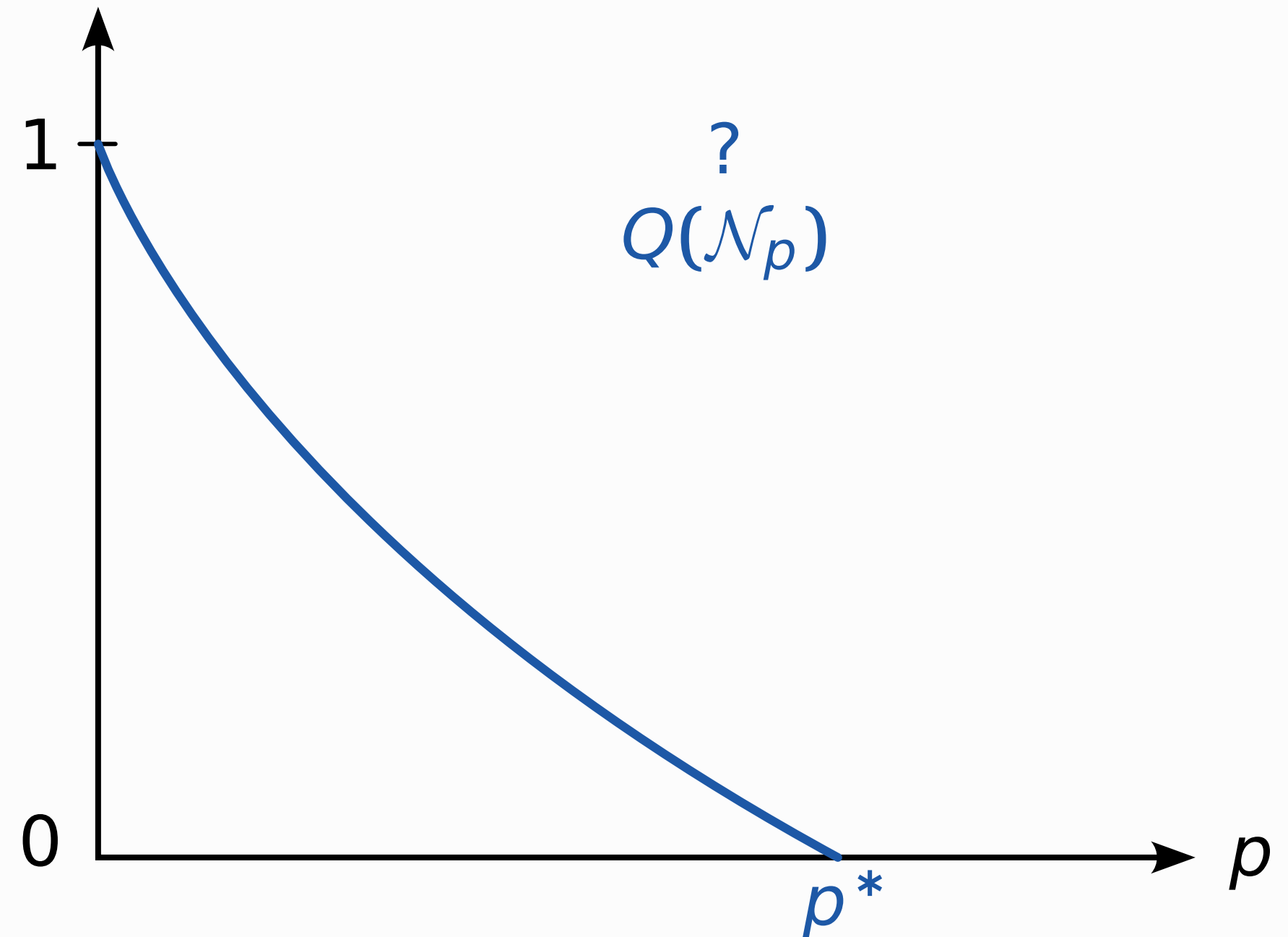


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$$p^* = \sup\{p : Q(\mathcal{N}_p) > 0\}$$



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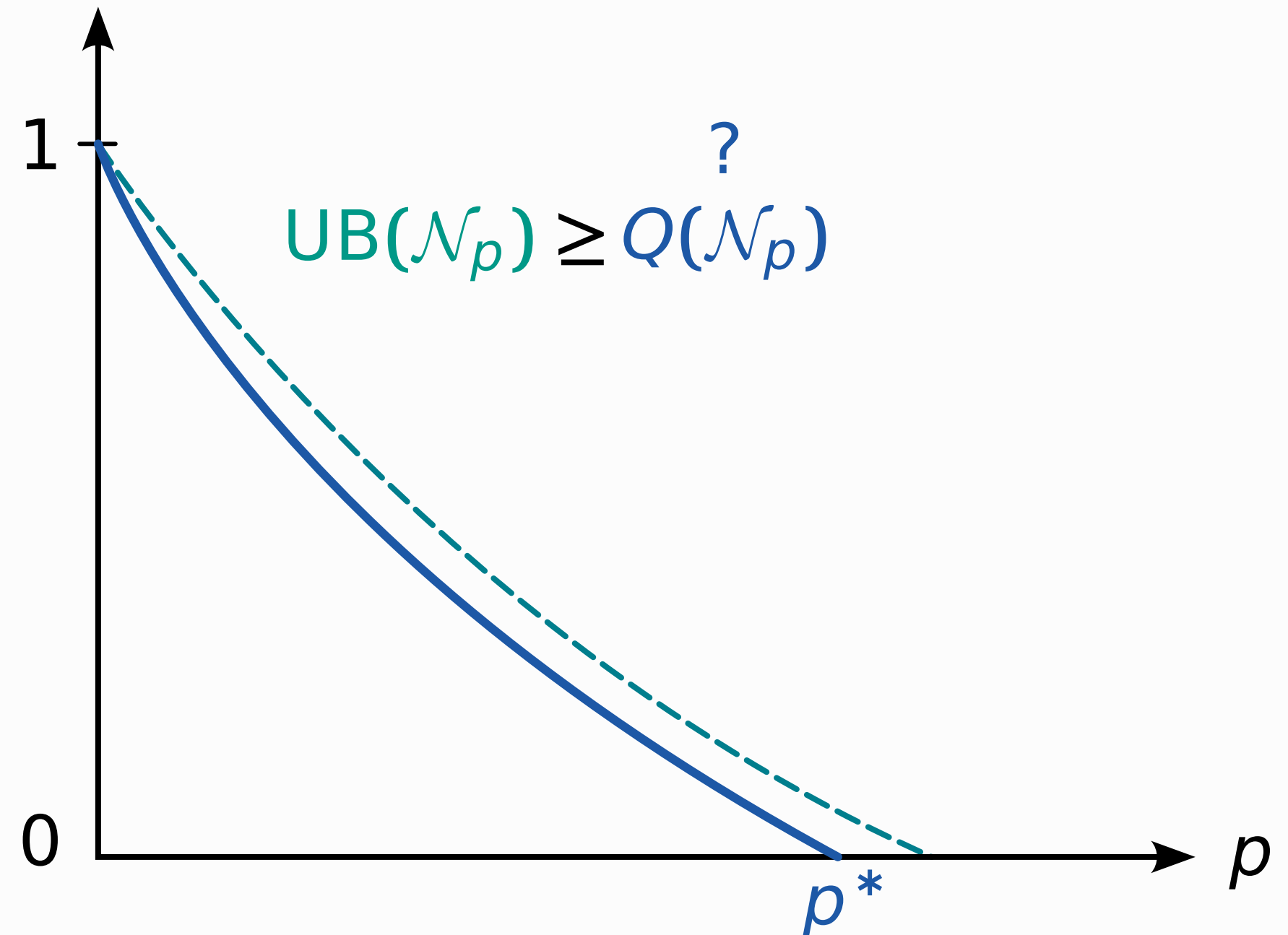


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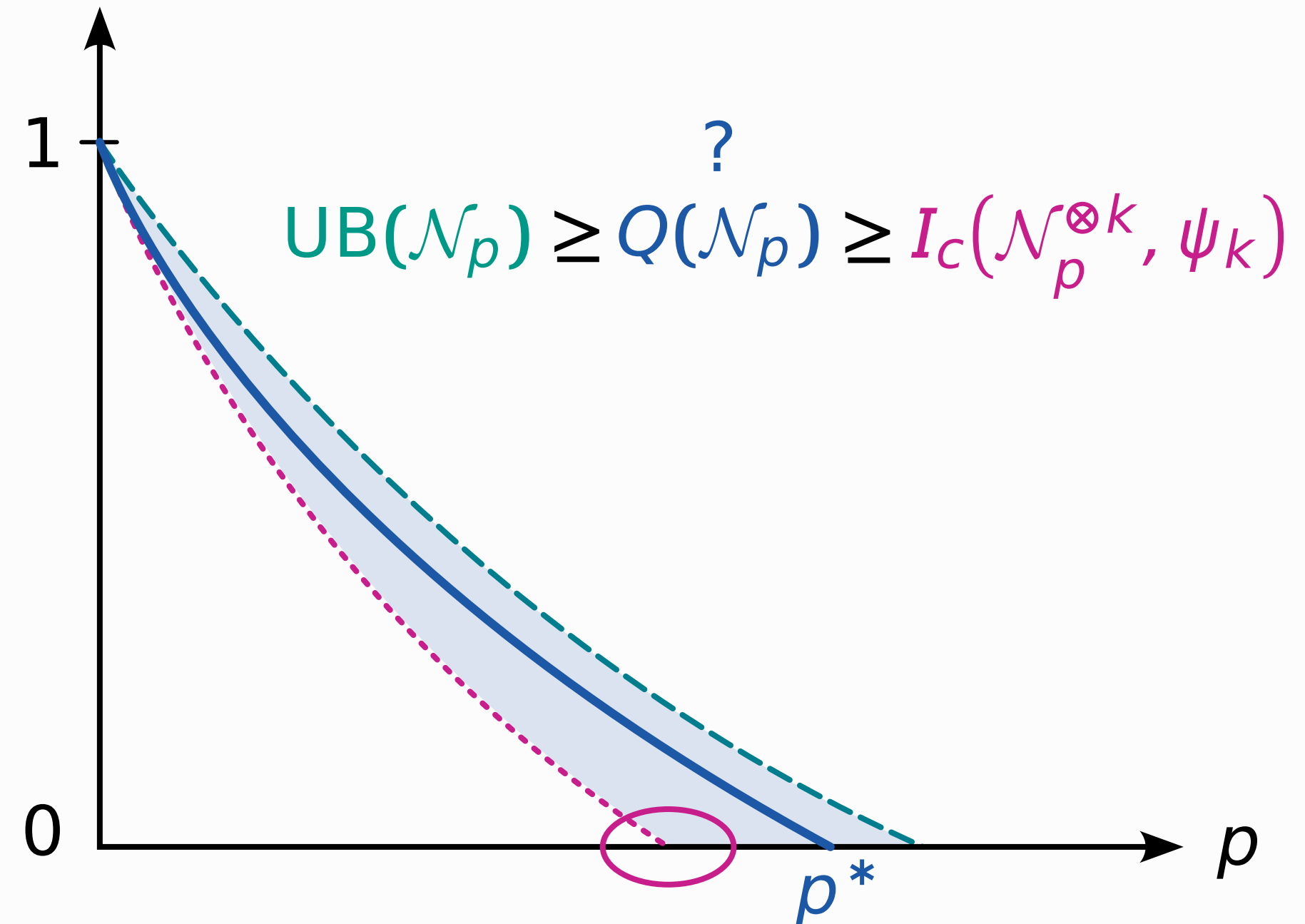
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Find ψ_k such that $I_c(\mathcal{N}_{p_0}^{\otimes k}, \psi_k) > 0$
for p_0 close to p^* .



Numerical studies



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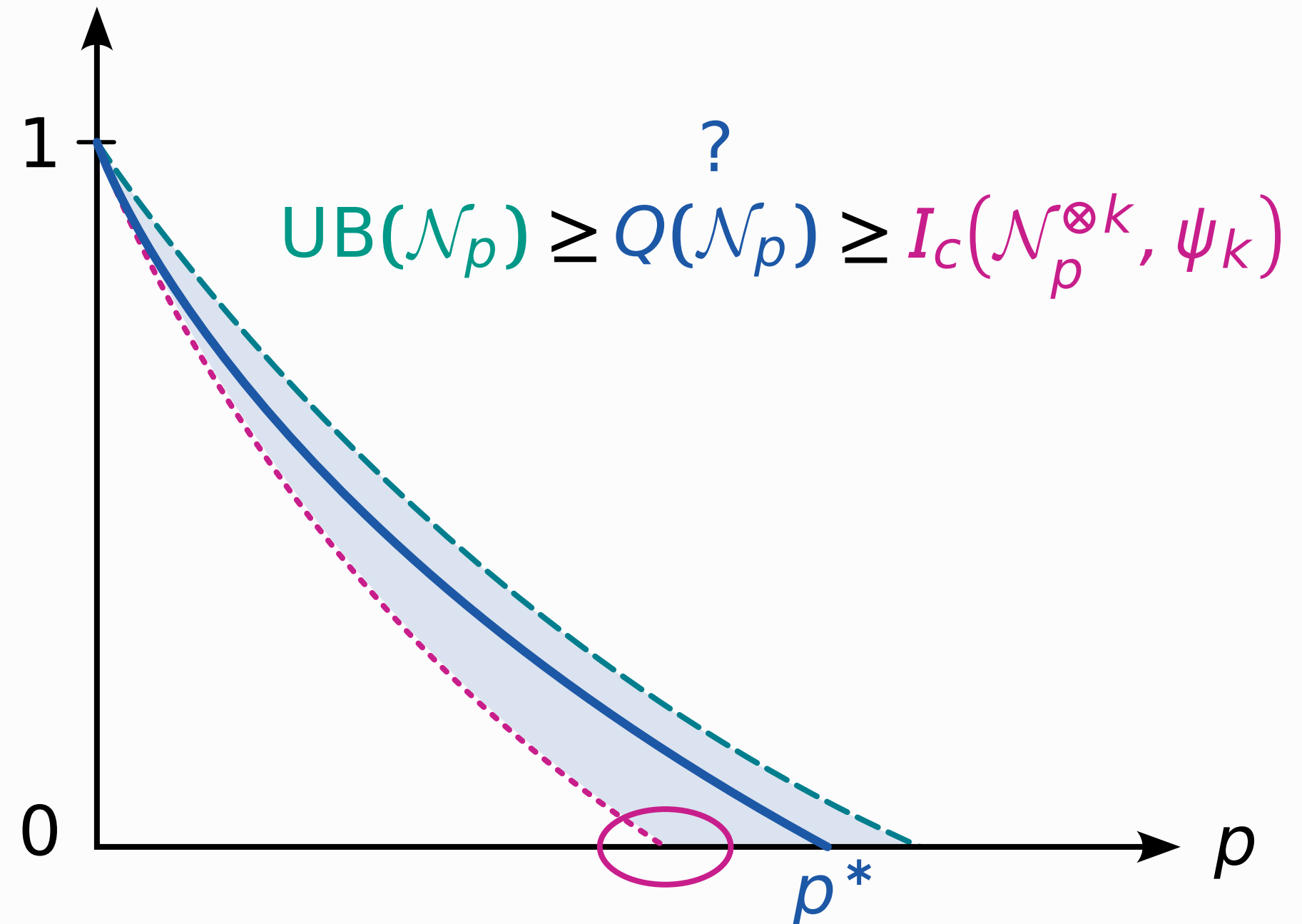
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$$\implies p^* \geq p_0$$



Two-Pauli channel



$$\mathcal{N}_p: \sigma \longmapsto (1-p)\sigma + \frac{p}{2}(X\sigma X + Z\sigma Z)$$

Two-Pauli channel



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Previously known bounds on quantum capacity threshold p^* :



Two-Pauli channel



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Previously known bounds on quantum capacity threshold p^* :

No-cloning bound: $p^* \leq 1/3$



Two-Pauli channel



$$\mathcal{N}_p: \sigma \longmapsto (1-p)\sigma + \frac{p}{2}(X\sigma X + Z\sigma Z)$$

Previously known bounds on quantum capacity threshold p^* :

Hashing bound: $p^* \geq 0.227 = \sup\{p : I_c(\mathcal{N}_p) > 0\}$



Two-Pauli channel



$$\mathcal{N}_p: \sigma \longmapsto (1-p)\sigma + \frac{p}{2}(X\sigma X + Z\sigma Z)$$

Previously known bounds on quantum capacity threshold p^* :

Fern, Whaley '08: $p^* \geq 0.228$ (over 10^9 channel qubits needed!)

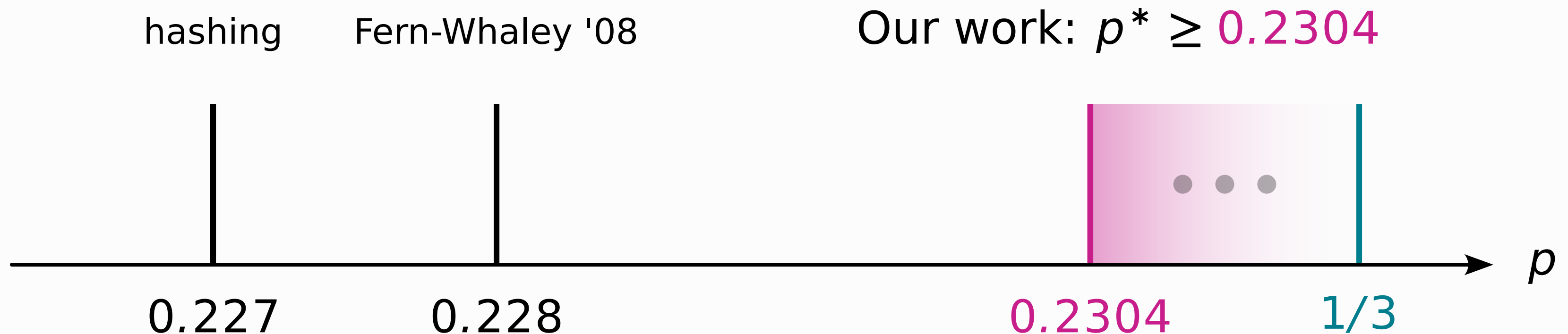


Two-Pauli channel



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Two-Pauli channel



Our work: Significantly improved lower bound $p^* \geq 0.2304$.

Strategy:

Optimize over states

$$\rho_{(n)} = \sum_i x_i \rho_i^{\otimes n}$$

and use rep-th. formula

to compute $I_c(\mathcal{N}_p^{\otimes n}, \rho_{(n)})$.

Two-Pauli channel



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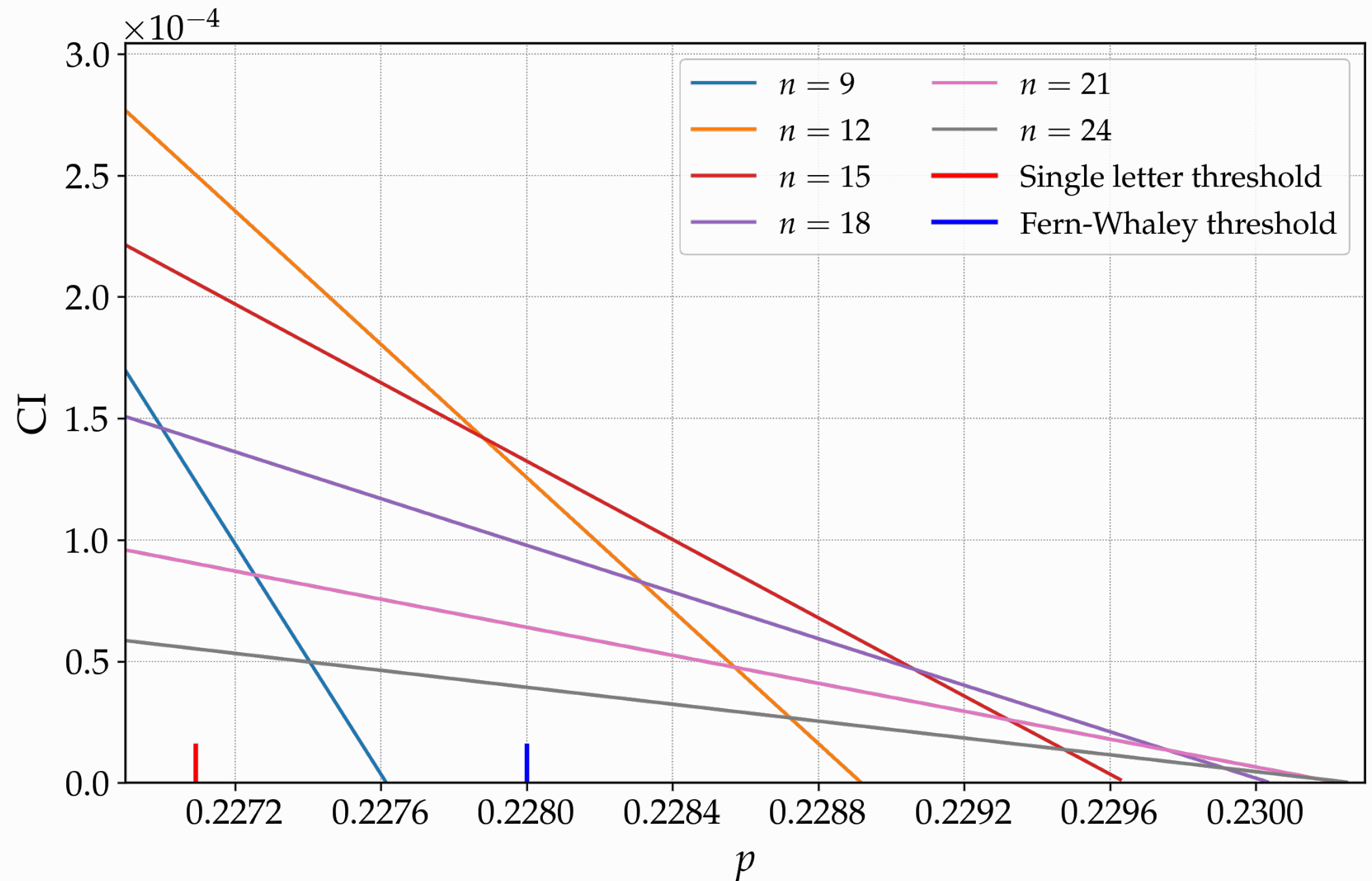
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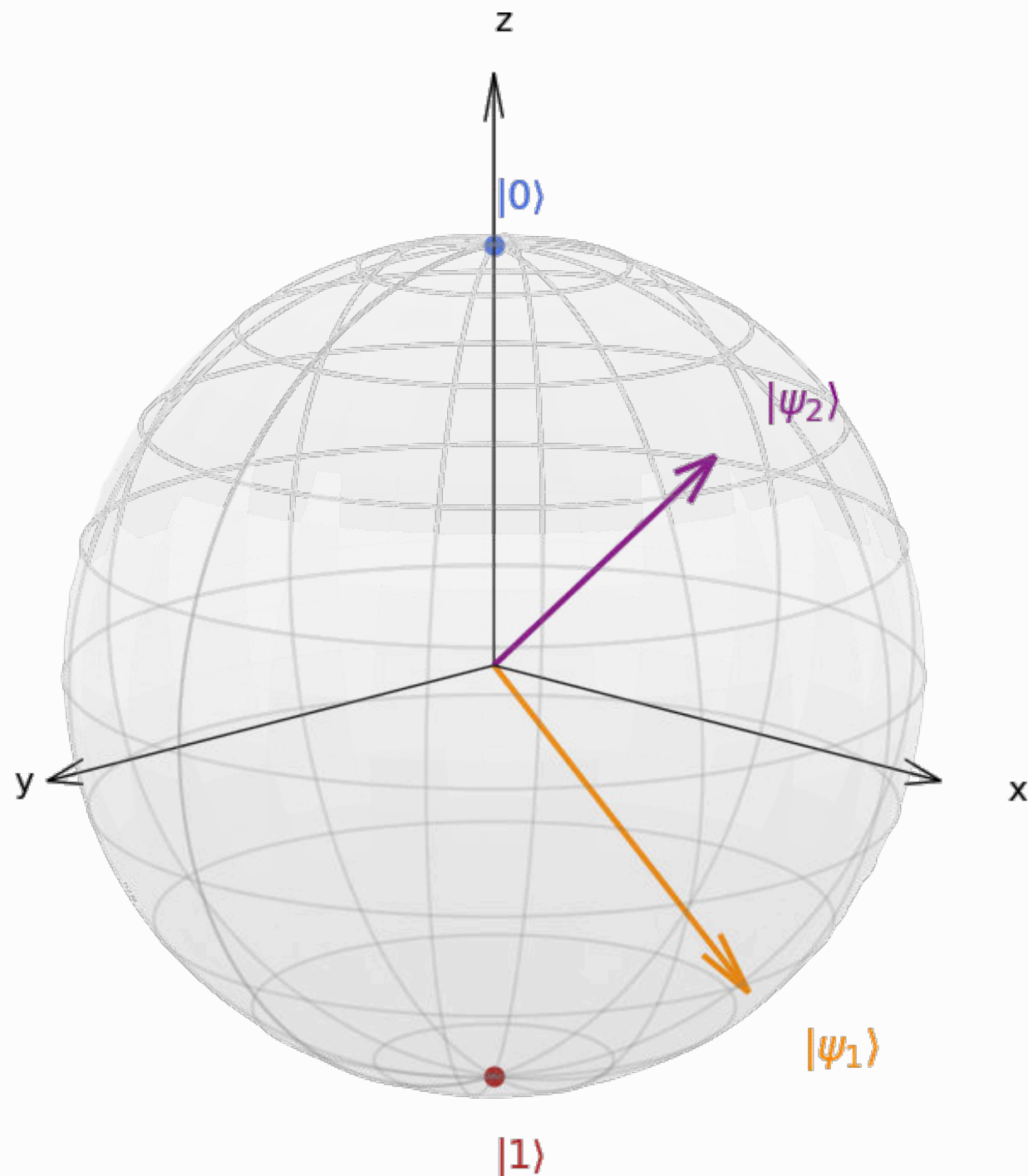


Two-Pauli channel



Optimal code: "non-orthogonal repetition code"

$$\rho(n) = \frac{1}{2} (\psi_1^{\otimes n} + \psi_2^{\otimes n}) \quad \text{with} \quad |\langle \psi_1 | \psi_2 \rangle| = 0.7645.$$

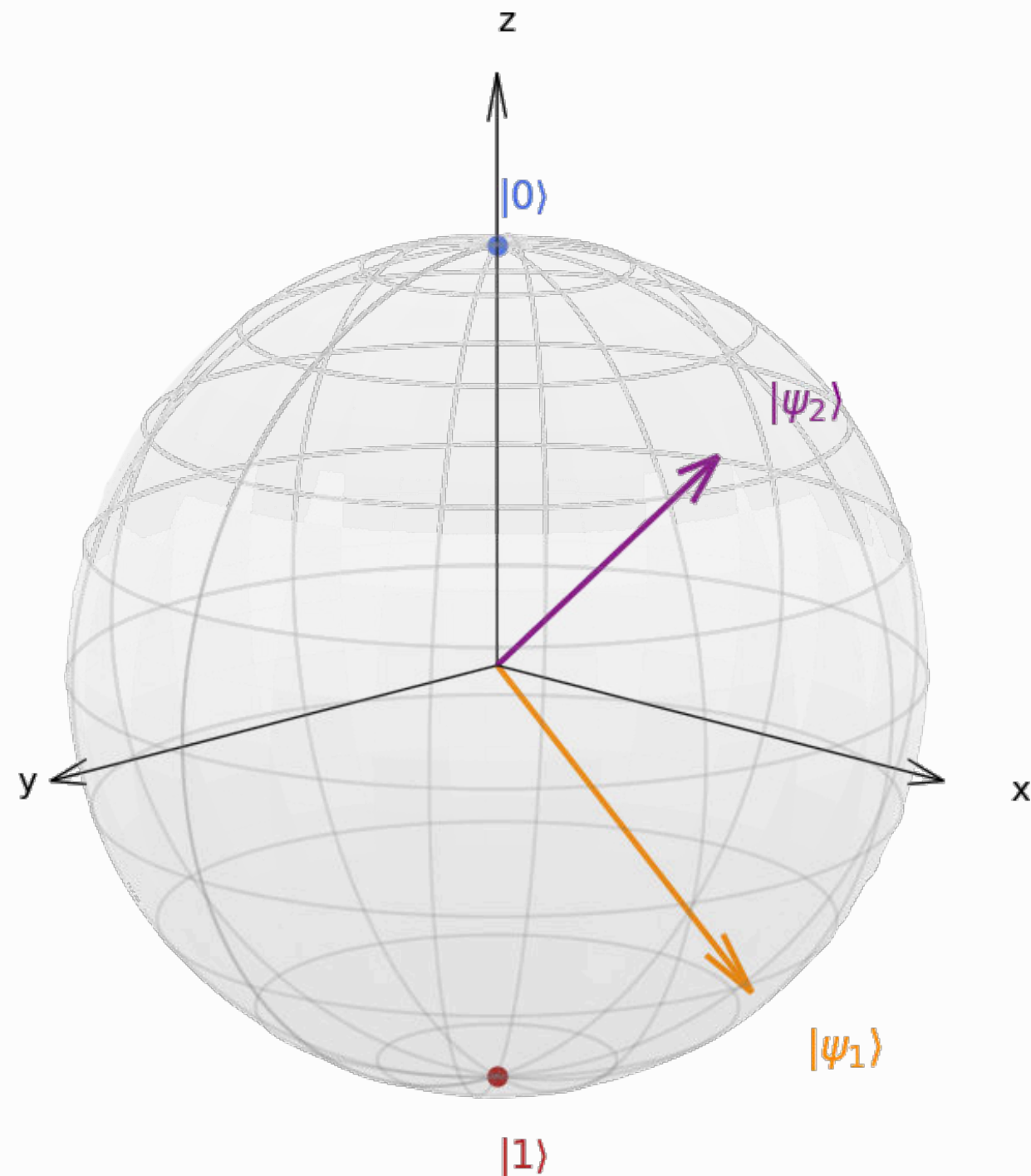


Two-Pauli channel



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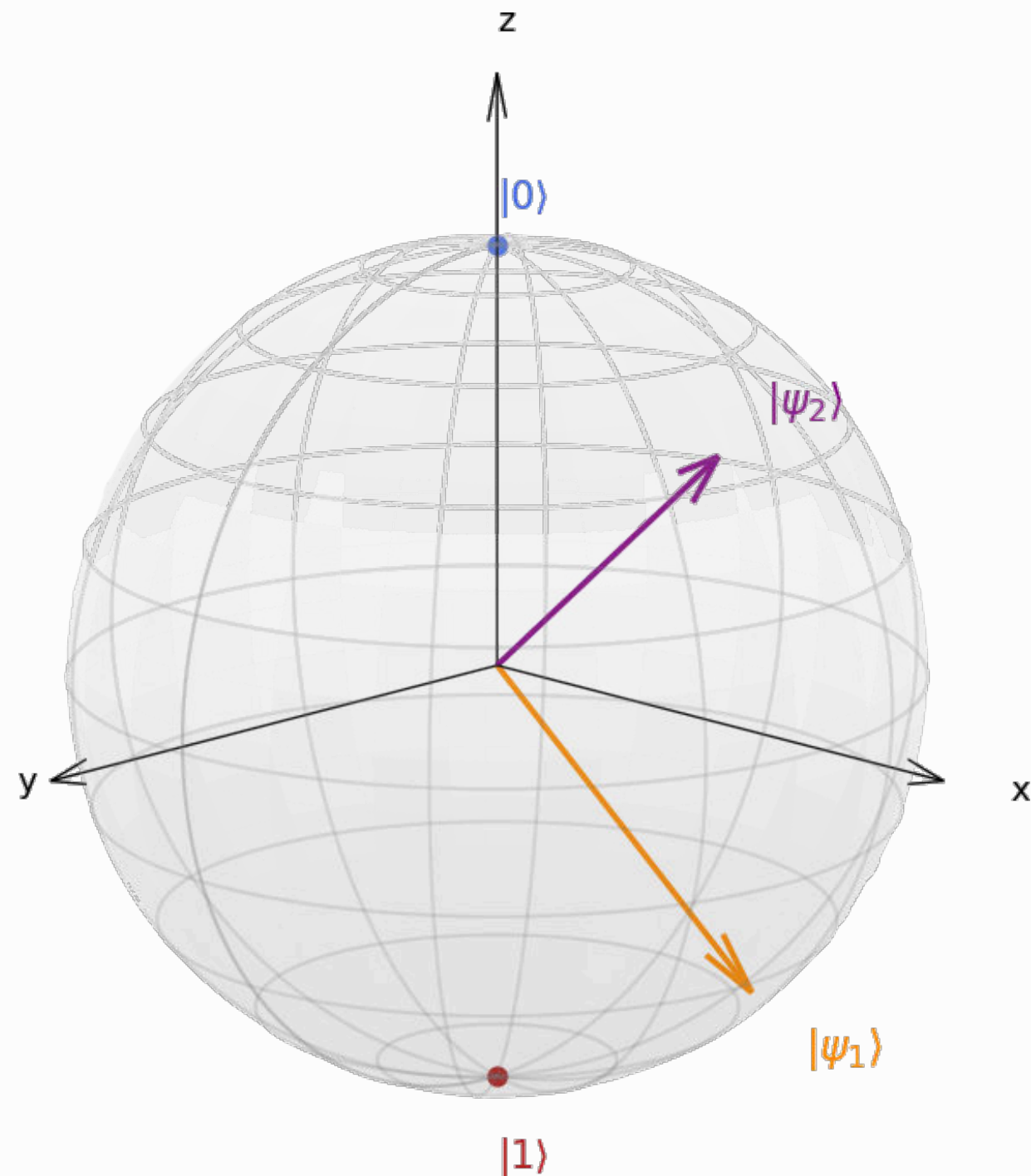
Usual orthogonal repetition codes with $\langle \psi_1 | \psi_2 \rangle = 0$ do not even beat the single-letter threshold!

Two-Pauli channel



Optimal code: "non-orthogonal repetition code"

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Other Pauli channels (e.g., BB84 channel) also benefit from non-orthogonal rep codes.

Representation-theoretic analysis in paper!



We use a permutation-invariant ansatz for quantum codes and tools from representation theory to improve achievable communication rates for many channels of interest:

- Pauli channels (e.g., two-Pauli, BB84)
- Generalized amplitude damping channel
- Dephasure channel
- Damping-dephasing channel



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Future work: Handle more general symmetry group to make code ansatz more flexible and tailored to specific noise models.

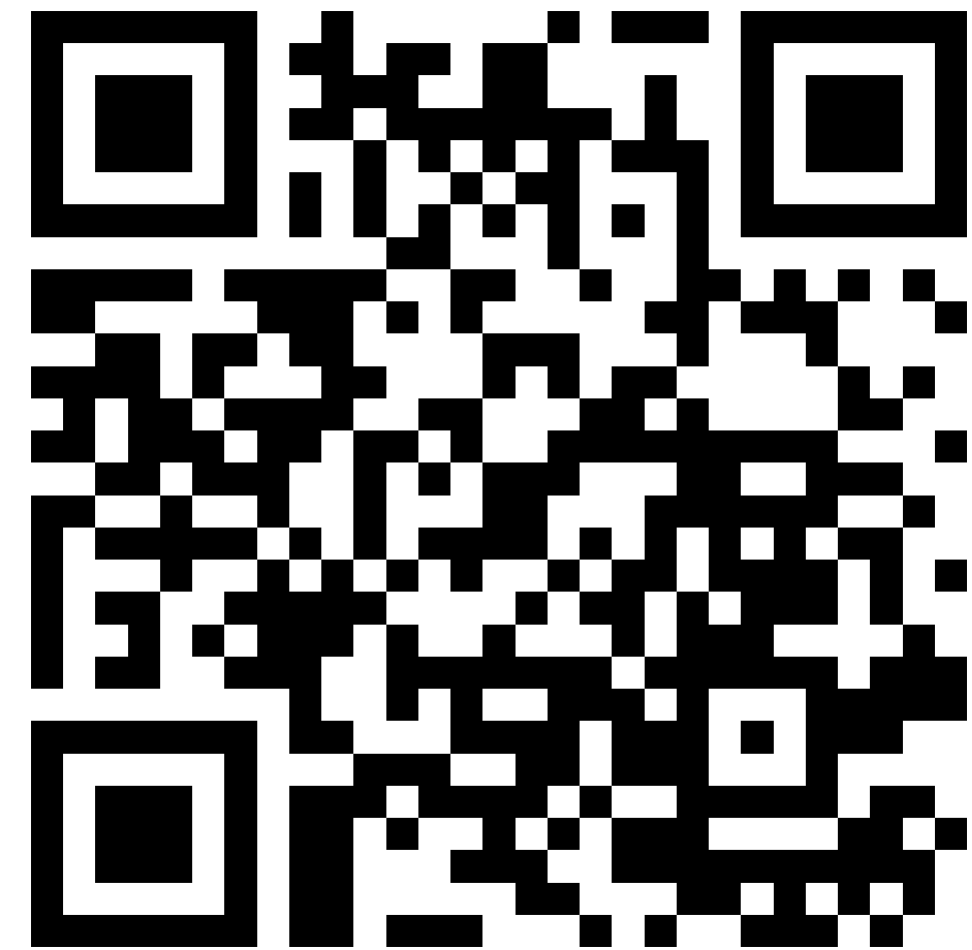
Links to paper and code



Paper



Code on GitHub



Thanks for your attention!