

# Bridging Quantum Information and Mathematical Physics

(August 14-16, 2024)

## Book of Abstracts

**Speaker:** Simon Becker

**Affiliation:** ETH Zurich

**Title:** Convergence rates for the Trotter-Kato splitting

**Abstract:** We study convergence rates of the Trotter-Kato splitting  $e^{A+L} = \lim_{n \rightarrow \infty} (e^{L/n} e^{A/n})^n$  in the strong operator topology. In the first part, we use complex interpolation theory to treat generators  $L$  and  $A$  of contraction semigroups on Banach spaces, with  $L$  relatively  $A$ -bounded. In the second part, we study unitary dynamics on Hilbert spaces and develop a new technique based on the concept of energy constraints. Our results provide a complete picture of the convergence rates for the Trotter splitting for all common types of Schrödinger and Dirac operators, including singular, confining and magnetic vector potentials, as well as molecular many-body Hamiltonians in dimension  $d = 3$ . Using the Brezis-Mironescu inequality, we derive convergence rates for the Schrödinger operator with  $V(x) = \pm |x|^{-a}$  potential. In each case, our conditions are fully explicit.

**Speaker:** Francesco Buscemi

**Affiliation:** Nagoya University

**Title:** Squashed non-Markovianity

**Abstract:** It is well known that convex combinations of Markov processes typically result in non-Markov ones. In this talk I will review some notions of (non-)Markovianity for quantum stochastic processes focusing in particular on a recent proposal to quantify information backflows after classical memories have been suitably squashed out. Such a “squashed” non-Markovianity, besides suggesting a notion of “genuine” or “causal” information revivals, is also able to resolve the problem of non-convexity, thus clarifying the role of non-Markovianity as a resource. The possibility of extending the same intuition to other non-convex resource theories is discussed. This is joint work with R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, and M.N. Bera. Preprint available as [arXiv:2405.05326](https://arxiv.org/abs/2405.05326).

**Speaker:** Ángela Capel

**Affiliation:** University of Cambridge

**Title:** Conditional Independence of 1D Gibbs States with Applications to Efficient Learning

**Abstract:** We show that spin chains in thermal equilibrium have a correlation structure in which individual regions are strongly correlated at most with their near vicinity. We quantify this with alternative notions of the conditional mutual information defined through the so-called Belavkin-Staszewski relative entropy. We prove that these measures decay superexponentially at every positive temperature, under the assumption that the spin chain Hamiltonian is translation-invariant. Using a recovery map associated with these measures, we sequentially construct tensor network approximations in terms of marginals of small (sublogarithmic) size. As a main application, we show that classical representations of the states can be learned efficiently from local measurements with a polynomial sample complexity. We also prove an approximate factorization condition for the purity of the entire Gibbs state, which implies that it can be efficiently estimated to a small multiplicative error from a small number of local measurements. The results extend from strictly local to exponentially-decaying interactions above a threshold temperature, albeit only with exponential decay rates. As a technical step of independent interest, we show an upper bound to the decay of the Belavkin-Staszewski relative entropy upon the application of a conditional expectation.

This is joint work with Paul Gondolf, Samuel Scalet, Alberto Ruiz de Alarcón and Álvaro Alhambra.

**Speaker:** Hao-Chung Cheng

**Affiliation:** National Taiwan University

**Title:** Joint State-Channel Decoupling

**Abstract:** In this work, we consider decoupling a bipartite quantum state via a general quantum channel. We propose a joint channel-state decoupling approach to obtain a one-shot error exponent without smoothing, in which trace distance is used to measure how good the decoupling is. The established exponent is expressed in terms of a sum of two sandwiched Rényi entropies, one quantifying the amount of initial correlation between the state and environment, while the other characterizing the effectiveness of the quantum channel. This gives an explicit exponential decay of the decoupling error in the whole achievable region.

**Speaker:** Matthias Christandl

**Affiliation:** University of Copenhagen

**Title:** Quantum Communication with Noisy Devices

**Abstract:** Designing encoding and decoding circuits to reliably send messages over many uses of a noisy channel is a central problem in communication theory. When studying the optimal transmission rates, it is usually assumed that these circuits can be implemented using noise-free gates. While this assumption is satisfied for classical machines in many scenarios, it is not expected to be satisfied in the near future for quantum machines where decoherence leads to faults in the quantum gates. As a result, fundamental questions regarding the practical relevance of quantum channel coding remain open. In the presented work, we initiate the study of these questions and show how techniques from quantum computation and communication can be combined to

arrive at a communication analog of the famous threshold theorem. The threshold theorem says that one can compute (and now also communicate), when the noise per gate is below a certain threshold. We expect our results to be relevant for satellite communication as well as the execution of quantum software on distributed quantum processor cores.

**Speaker:** Gilad Gour

**Affiliation:** Technion - Israel Institute of Technology

**Title:** Quantum Additivity Implies Negativity

**Abstract:** In the domain of states, the concept of majorization serves as a fundamental tool for comparing the uncertainty inherent in both classical and quantum systems. In this talk I will introduce a rigorous framework for assessing the uncertainty in both classical and quantum channels. By employing a specific class of superchannels, we introduce and elucidate three distinct approaches to channel majorization: constructive, axiomatic, and operational. Intriguingly, these methodologies converge to a consistent ordering. This convergence not only provides a robust basis for defining entropy functions for channels but also clarifies the interpretation of entropy in this broader context. Most notably, our findings reveal that any viable entropy function for quantum channels must assume negative values, thereby challenging traditional notions of entropy. This is joint work with Doyeong Kim, Takla Nateebon, Guy Shemesh, and Goni Yoeli. Preprint available at <https://arxiv.org/abs/2406.13823>

**Speaker:** Pawel Horodecki

**Affiliation:** University of Gdańsk, Gdańsk University of Technology

**Title:** Unbounded quantum advantage. An instance with low number of inputs

**Abstract:** Communication complexity of relations (CCR) based on graph clique labelling is investigated and its variant called strong-CCR is defined. For the latter quantum advantage with unbounded gap between classical and quantum cost of communication is identified. Unlike in some previous scenarios in the field the gap is of the order of the size of the inputs. The lower bound for shared randomness to close the classical-quantum gap is given in terms of orthogonal arrays. Some open questions are discussed.

**Speaker:** Min-Hsiu Hsieh

**Affiliation:** Hon Hai Research Institute

**Title:** Constant-Overhead Magic State Distillation

**Abstract:** Magic state distillation is a crucial yet resource-intensive process in fault-tolerant quantum computation. The number of noisy input magic states required for each output magic state with an error rate of  $\varepsilon$  is referred to as the protocol's overhead, which is known to grow as  $O(\log^\gamma(1/\varepsilon))$ . Therefore, discovering protocols with small exponents  $\gamma$  is highly desirable. In this work, we construct protocols for qubit magic state distillation achieving  $\gamma = 0$ , meaning an  $O(1)$  overhead, for the first time. Protocols with  $\gamma < 1$  have been achieved twice before — first in the work of Hastings and

Haah, giving qubit schemes with  $\gamma \approx 0.678$ , and then in the work of Krishna and Tillich, which gave  $\gamma \rightarrow 0$  in the limit of infinitely large-dimensional qudits. Whereas these previous works used Reed-Muller codes and Reed-Solomon codes, respectively, we use algebraic geometry codes to achieve  $\gamma = 0$  for qubits. In this way, we explicitly construct asymptotically good quantum codes on 1024-dimensional qudits that support a transversal implementation of a logical gate in the third level of the Clifford hierarchy. We show that this code can be interpreted as a code on groups of 10 qubits. The gate that is naturally distilled is then a 10-qubit gate in the third level of the Clifford hierarchy, but we show that there is a conversion to the conventional magic states for a control-control-Z (CCZ) gate and a T gate on qubits, up to only a constant loss of the overhead. The use of asymptotically good codes enables not only  $\gamma \rightarrow 0$  but also  $\gamma = 0$ , constant-overhead magic state distillation, in the following sense. There exist constants  $\Omega$  and  $p_{\text{th}}$  such that, for any target error rate  $\varepsilon$ , there is some large enough  $n$  such that if we are given a supply of  $n$  magic states of error rate less than a threshold  $p_{\text{th}}$ , we can produce  $k$  magic states of error rate below  $\varepsilon$  within an overhead  $n/k \leq \Omega$ .

**Speaker:** Zahra Khanian

**Affiliation:** Technical University of Munich

**Title:** Quantum Reverse Shannon Theorem Revisited

**Abstract:** Reverse Shannon theorems concern the use of noiseless channels to simulate noisy ones. This is dual to the usual noisy channel coding problem, where a noisy (classical or quantum) channel is used to simulate a noiseless one. The Quantum Reverse Shannon Theorem and the resource trade-offs for simulating quantum channels are extensively studied by Bennett and co-authors in their seminal work published in [IEEE-IT/2014]. They present two distinct theorems, each tailored to classical and quantum channel simulations respectively, explaining the fact that these theorems remain incomparable due to the fundamentally different nature of correlations they address. The authors leave as an open question the challenge of formulating a unified theorem that could encompass the principles of both and unify them. We unify these two theorems into a single, comprehensive theorem, extending it to the most general case by considering correlations with a general mixed-state reference system. Furthermore, we unify feedback and non-feedback theorems by simulating a general side information system at the encoder side. In this general model, we consider noiseless quantum communication and entanglement as resources, and explore the optimal trade-off between these resources.

**Speaker:** Ludovico Lami

**Affiliation:** University of Amsterdam

**Title:** A solution of the generalised quantum Stein's lemma

**Abstract:** We solve the generalised quantum Stein's lemma, proving that the Stein exponent associated with entanglement testing, namely, the quantum hypothesis testing task of distinguishing between  $n$  copies of an entangled state and a generic separable state, equals the regularised relative entropy of entanglement. Not only does this

determine the ultimate performance of entanglement testing, but it also establishes the reversibility of all quantum resource theories under asymptotically resource non-generating operations, with the regularised relative entropy of resource governing the asymptotic transformation rate between any two quantum states. To solve the problem we introduce two techniques. The first is a procedure that we call “blurring”, which, informally, transforms a permutationally symmetric state by making it more evenly spread across nearby type classes. In the fully classical case, blurring alone suffices to prove the generalised Stein's lemma. To solve the quantum problem, however, it does not seem to suffice. Our second technical innovation, therefore, is to perform a second quantisation step to lift the problem to an infinite-dimensional bosonic quantum system; we then solve it there by using techniques from continuous-variable quantum information. Rather remarkably, the second-quantised action of the blurring map corresponds to a pure loss channel. A careful examination of this second quantisation step is the core of our quantum solution.

**Speaker:** Felix Leditzky

**Affiliation:** University of Illinois Urbana-Champaign

**Title:** On the duality of teleportation and dense coding

**Abstract:** Quantum teleportation is a quantum communication primitive that allows a long-distance quantum channel to be built using pre-shared entanglement and one-way classical communication. However, the quality of the established channel crucially depends on the quality of the pre-shared entanglement. In this work, we revisit the problem of using noisy entanglement for the task of teleportation. We first show how this problem can be rephrased as a state discrimination problem. In this picture, a quantitative duality between teleportation and dense coding emerges in which every Alice-to-Bob teleportation protocol can be repurposed as a Bob-to-Alice dense coding protocol, and the quality of each protocol can be measured by the success probability in the same state discrimination problem. One of our main results provides a complete characterization of the states that offer no advantage in one-way teleportation protocols over classical states, thereby offering a new and intriguing perspective on the long-standing open problem of identifying such states. This also yields a new proof of the known fact that bound entangled states cannot exceed the classical teleportation threshold. Moreover, our established duality between teleportation and dense coding can be used to show that the exact same states are unable to provide a non-classical advantage for dense coding as well. We also discuss the duality from a communication capacity point of view, deriving upper and lower bounds on the accessible information of a dense coding protocol in terms of the fidelity of its associated teleportation protocol. A corollary of this discussion is a simple proof of the previously established fact that bound entangled states do not provide any advantage in dense coding.

**Speaker:** Milán Mosonyi

**Affiliation:** Budapest University of Technology and Economics

**Title:** Super-exponential distinguishability of correlated quantum states

**Abstract:** In the problem of asymptotic binary i.i.d. state discrimination, the optimal asymptotics of the type I and the type II error probabilities is in general an exponential decrease to zero as a function of the number of samples; the set of achievable exponent pairs is characterized by the quantum Hoeffding bound theorem. A super-exponential decrease for both types of error probabilities is only possible in the trivial case when the two states are orthogonal, and hence can be perfectly distinguished using only a single copy of the system. In this talk it is shown that a qualitatively different behaviour can occur when there is correlation between the samples. Namely, gauge-invariant and translation-invariant quasi-free states on the algebra of the canonical anti-commutation relations can be used to exhibit pairs of states on an infinite spin chain with the properties that a) all finite-size restrictions of the states have invertible density operators, and b) the type I and the type II error probabilities both decrease to zero at least with the speed  $\exp(-nc \log n)$  with some positive constant  $c$ , i.e., with a super-exponential speed in the sample size  $n$ .

Based on <https://link.springer.com/article/10.1007/s11005-022-01620-4>.

**Speaker:** Bruno Nachtergaele

**Affiliation:** University of California, Davis

**Title:** Analysis of quantum states of matter by classic statistical mechanics

**Abstract:** I review old and new results about quantum states of matter obtained using techniques borrowed from classical statistical mechanics. Several first-of-their-kind results have been proved this way. We argue that beyond yielding interesting new results, the application of statistical mechanics ideas to the ground state problem of quantum many-body systems have led to breakthroughs in our understanding of the principles governing these systems and topological states of matter.

**Speaker:** Renato Renner

**Affiliation:** ETH Zurich

**Title:** Almost-IID information theory

**Abstract:** Bruno de Finetti cautioned that the “most common and misleading error” in probability theory is treating the IID assumption as fundamental. However, recent theorems, notably generalisations of a theorem by de Finetti himself, indicate that an “almost-IID” structure can emerge from well-justified operational symmetry assumptions. An almost-IID state is characterised by having most subsystems in an IID state. This prompts the question: Are almost-IID resources as effective as perfect IID resources for information-processing tasks? In my presentation, I will explore this question and provide some preliminary answers.

**Speaker:** Cambyse Rouzé

**Affiliation:** Inria - Institut Polytechnique de Paris

**Title:** Quantum information perspectives on Gibbs sampling

**Abstract:** Markov Chain Monte Carlo methods are one of the go-to tools for sampling from classical Gibbs states of spin systems. They exhibit efficiency in practice and are

also provably efficient in some cases, such as at high temperatures. These methods traditionally rely on “local updates”, where a few spins are flipped based on local energy changes, a feature crucial for both practical implementation and theoretical analysis. One rigorous method to get optimal bounds on the runtime of MCMC algorithms for Gibbs sampling is via local-to-global strategies involving the establishment of tight entropic inequalities. However, extending such reasoning to quantum systems has been a formidable challenge. In this talk, I will present some recent advances in the finding of realistic conditions under which fast convergence of certain quantum Gibbs samplers can be established.

**Speaker:** Reinhard Werner

**Affiliation:** Leibniz Universität Hannover

**Title:** Quantum gases via the Fredholm determinant

**Abstract:** The grand canonical theory of Bose and Fermi gases is basic textbook material, usually based on the pre-quantum counting rules for occupation numbers. This loses some of the non-commutative aspects of the theory, for example easy access to the position distribution of Bose particles. I will describe a simple approach amending this, which requires only one technical detail: the theory of infinite dimensional “Fredholm” determinants, which occur in the basic formula for general quasifree partition functions. This approach should be “standard”, and I’d be happy to learn from the audience where in the literature it can maybe already be found.

**Speaker:** Mark M. Wilde

**Affiliation:** Cornell University

**Title:** Barycentric bounds on the error exponents of quantum hypothesis exclusion

**Abstract:** Quantum state exclusion is an operational task that has significance in studying foundational questions related to interpreting quantum theory. In such a task, one is given a system whose state is randomly selected from a finite set, and the goal is to identify a state from the set that is *\emph{not}* the true state of the system. An error, i.e., an unsuccessful exclusion, occurs if and only if the state identified is the true state. In this paper, we study the optimal error probability of quantum state exclusion and its error exponent — the rate at which the error probability decays asymptotically — from an information-theoretic perspective. Our main finding is a single-letter upper bound on the error exponent of state exclusion given by the multivariate log-Euclidean Chernoff divergence, and we prove that this improves upon the best previously known upper bound from [Mishra *et al.*, *Letters in Mathematical Physics* **114**, 76 (2024)]. We also extend our analysis to the more complicated task of quantum channel exclusion, and we establish a single-letter and efficiently computable upper bound on its error exponent, even assuming the use of adaptive strategies. We derive both upper bounds, for state and channel exclusion, based on one-shot analysis and formulate them as a type of multivariate divergence measure called a barycentric Chernoff divergence. Moreover, our result on channel exclusion has implications in two important special cases. First, for the special case of two hypotheses, our upper bound provides the first known

efficiently computable upper bound on the error exponent of symmetric binary channel discrimination. Second, for the special case of classical channels, we show that our upper bound is achievable by a nonadaptive strategy, thus solving the exact error exponent of classical channel exclusion and generalizing a similar result on symmetric binary classical channel discrimination from [Hayashi, IEEE Transactions on Information Theory 55, 3807 (2009)]. Joint work with Kaiyuan Ji, Hemant Mishra, and Milan Mosonyi and available as <https://arxiv.org/abs/2407.13728>.

**Speaker:** Andreas Winter

**Affiliation:** Universitat Autònoma de Barcelona

**Title:** Hidden Markov models - classical and quantum mechanisms, and beyond

**Abstract:** Among the stationary time series of events (in discrete time steps and with signals from a discrete alphabet), those generated as a function of a hidden Markov chain have a special place.

Here, we are studying the case that the hidden system is quantum mechanical, giving rise to  $C^*$ -finitely correlated states [Fannes/Nachtergaele/Werner, CMP 1992] on a commutative chain algebra, which we call hidden quantum Markov models; and even more generally finitely correlated states, corresponding to a general probabilistic theory (GPT) memory. The latter case is entirely described in terms of the rank of the so-called Hankel matrix of the process, and equivalently an associated canonical finite-dimensional vector space with associated positive cone preserved under the hidden dynamics of the model. For the quantum case, we describe the structure of the possible GPTs via semidefinite representable (SDR) cones. It turns out that these GPTs are all finitely presented operator systems, i.e. induced subspaces of quotients of  $B(H)$  for a finite-dimensional Hilbert space  $H$ . Unlike operator systems, for which complete positivity can be very hard to decide, the SDR models come with a subset of the completely positive maps, which is itself an SDR cone. I will describe a geometric approach, through the study of the positive cone of any GPT giving rise to a given process, which can sometimes show that certain processes with hidden quantum Markov model have no finite classical memory model.

I will then describe the first known example of a process generated via a finite-dimensional GPT as the hidden system, which however cannot be reproduced by any hidden classical or quantum Markov model with finite state space, answering a question posed by Fannes, Nachtergaele and Werner. Processes generated via a finite-dimensional GPT which cannot be reproduced by a hidden classical Markov chain had been known before, but the examples exhibited there turn out to be generated by hidden quantum Markov models.