Recap: Werner states

Definition

A bipartite quantum state ρ_{AB} on $\mathbb{C}^d \otimes \mathbb{C}^d$ is called *Werner state* if

$$\rho_{AB} = (U \otimes U)\, \rho_{AB}\, (U \otimes U)^\dagger \quad \text{for all } U \in \mathcal{U}_d.$$

Two-qubit Werner states

Every quantum state ρ_{AB} on $\mathbb{C}^2 \otimes \mathbb{C}^2$ satisfying $\rho_{AB} = (U \otimes U)\, \rho_{AB}\, (U \otimes U)^\dagger$ for all $U \in \mathcal{U}_2$ can be written in terms of a single parameter $x \in [-1, 1]$ as

$$\rho_{AB} = \frac{2 - x}{6} \mathbb{1}_{AB} + \frac{2x - 1}{6} \mathbb{F}_{AB}.$$

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Haar measure

There exists a unique uniform probability measure $\mathrm{d}U$ on the unitary group $\mathcal{U}_d$ called *Haar measure*, satisfying

(i) Normalization: $\int_{\mathcal{U}_d} \mathrm{d}U = 1$;

(ii) Left- and right-invariance: $\int_{\mathcal{U}_d} \mathrm{d}U\, f(U) = \int_{\mathcal{U}_d} \mathrm{d}U\, f(VU) = \int_{\mathcal{U}_d} \mathrm{d}U\, f(UV)$ for any $V \in \mathcal{U}_d$.

Twirling

The twirling map

$$\mathcal{T}(X_{AB}) = \int_{\mathcal{U}_d} \mathrm{d}U\, (U \otimes U) X_{AB} (U \otimes U)^\dagger$$

maps every bipartite quantum state ρ_{AB} onto a Werner state with $x = \mathrm{tr}(\mathbb{F}_{AB}\rho_{AB})$.

Entanglement of two-qubit Werner states

The Werner state $\rho_{AB} = \frac{2-x}{6}\mathbb{1}_{AB} + \frac{2x-1}{6}\mathbb{F}_{AB}$ is entangled if and only if $x < 0$.