End-of-term presentations

Timeline

- **Thursday, Oct 16:** Choose group and topic
  (from list posted below, on course website, and on Campuswire).
- **Thursday, Nov 13:** Submit a 1-page talk outline to me.
- **December:** Presentations (exact dates TBD, but prepare to be ready by Dec 2).
  10-15min speaking time per group member.

List of topics for end-of-term presentations

- Character theory, Schur polynomials, symmetric functions
- Testing separability
- NPT bound entanglement and Werner states
- Unitary designs
- Pseudorandomness and pseudoentanglement
- Post-selection technique and applications in QKD and channel coding
- Weingarten calculus and free probability
- More De Finetti theorems
- Commutant of the Clifford group
- Studying entanglement in tripartite systems using Schur-Weyl duality
- Consensus for Quantum Networks: Symmetry From Gossip Interactions (already assigned)

Pointers to literature will be provided.

Recap: Commutant theorem

Representations of the unitary and general linear group

A representation of $\mathcal{U}(V)$ is irreducible if and only if the corresponding representation of $\mathrm{GL}(V)$ is irreducible.

Commutants of S_n and GL(V)

S_n and $\mathrm{GL}(V)$ span each other's commutants in $\mathrm{End}(V^{\otimes n})$.

- Every operator $X \in \mathrm{End}(V^{\otimes n})$ commuting with all $M^{\otimes n}$ ($M \in \mathrm{End}(V)$) can be written as

$$X = \sum_{\pi \in S_n} c_\pi \varphi(\pi) \quad \text{for suitable } c_\pi \in \mathbb{C}.$$

- Every permutation-invariant operator $Y \in \mathrm{End}(V^{\otimes n})$ can be written as

$$Y = \sum_i a_i X_i^{\otimes n} \quad \text{for suitable } a_i \in \mathbb{C} \text{ and } X_i \in \mathrm{End}(V).$$

Recap: Schur-Weyl duality

Schur-Weyl duality

Let $V = \mathbb{C}^d$ and $(\varphi, V^{\otimes n})$ and $(\omega, V^{\otimes})$ be the tensor representations of S_n and $\mathrm{GL}(V)$ defined before. As a representation of $S_n \times \mathrm{GL}(V)$, the space $V^{\otimes n}$ decomposes as

$$V^{\otimes n} = \bigoplus_\lambda V_\lambda \otimes U_\lambda,$$

where $(\varphi_\lambda, V_\lambda)$ and $(\omega_\lambda, U_\lambda)$ are inequivalent irreducible representations of S_n and $\mathrm{GL}(V)$, respectively, and

$$\varphi(\pi) = \bigoplus_\lambda \varphi_\lambda(\pi) \otimes \mathbb{1}_{U_\lambda} \qquad \text{for } \pi \in S_n$$

$$\omega(g) = \bigoplus_\lambda \mathbb{1}_{V_\lambda} \otimes \omega_\lambda(g) \qquad \text{for } g \in \mathrm{GL}(V).$$

The same assertion holds when $\mathrm{GL}(V)$ is replaced with $\mathcal{U}(V)$.

Recap: Consequences of Schur-Weyl duality

A permutation-invariant state is block-diagonal with respect to the Schur-Weyl decomposition:

$$\rho \cong \bigoplus_\lambda \mathbb{1}_{V_\lambda} \otimes \rho_\lambda \quad \text{for some } \rho_\lambda \in \mathrm{End}(U_\lambda^d).$$

By the same argument, a state commuting with all unitaries of the form $U^{\otimes n}$ with $U \in \mathcal{U}_d$ (a so-called **multipartite Werner state**) is of the form

$$\rho \cong \bigoplus_\lambda \tilde{\rho}_\lambda \otimes \mathbb{1}_{U_\lambda^d} \quad \text{for some } \tilde{\rho}_\lambda \in \mathrm{End}(V_\lambda).$$

If a state has both symmetries, $[\rho, \varphi(\pi)] = 0$ for all $\pi \in S_n$ and $[\rho, U^{\otimes n}] = 0$ for all $U \in \mathcal{U}_d$, then both statements above apply, and we have

$$\rho \cong \bigoplus_\lambda c_\lambda \mathbb{1}_{V_\lambda} \otimes \mathbb{1}_{U_\lambda^d} \quad \text{for some } c_\lambda \in \mathbb{C}.$$

Since this is a block decomposition, ρ is positive semidefinite iff $c_\lambda \geq 0$ for all λ.