Recap: De Finetti theorem for pure symmetric states

Formula for the symmetric subspace projector

$$\Pi_{\text{sym}} = \binom{n+d-1}{n} \int_{\mathcal{D}_1(\mathbb{C}^d)} \mathrm{d}\phi \, |\phi\rangle\langle\phi|^{\otimes n}.$$

De Finetti theorem for pure symmetric states

Let $\mathcal{H}_{A_i} \cong \mathbb{C}^d$ and $|\psi\rangle_{A_1 \cdots A_n} \in \mathrm{Sym}^n(\mathbb{C}^d)$. Then for any $k < n$,

$$D\left(\psi_{A_1 \cdots A_k}, \int_{\mathcal{D}_1(\mathbb{C}^d)} \mathrm{d}\phi \, p_\psi(\phi) |\phi\rangle\langle\phi|^{\otimes k}\right) \leq \sqrt{\frac{dk}{n-k}},$$

where $p_\psi(\phi)$ is a probability density that depends on $|\psi\rangle$.

Proof idea

Treat $\left\{\binom{n+d-1}{n}|\phi\rangle\langle\phi|^{\otimes n}\right\}_\phi$ as a continuous POVM on $\mathrm{Sym}^n(\mathbb{C}^d)$ and measure the $n-k$ extra systems with it. The resulting post-measurement state is separable on average.