Recap: Extendibility of quantum states

Definition

A bipartite state ρ_{AB} is called k-extendible if there exists a state $\rho_{AB_1\cdots B_k}$ (called k-extension) where each $B_i \cong B$ is a copy of the B-system and

$$\rho_{AB_i} = \mathrm{tr}_{B_1\cdots B_{i-1}B_{i+1}\cdots B_k}\rho_{AB_1\cdots B_k} = \rho_{AB} \quad \text{for all } i = 1,\ldots,k.$$

Pure entangled states are not even 2-extendible.

Extendibility hierarchy

Every k-extendible state is also k'-extendible for $k' \leq k$.

Proposition

Separable states are ∞-extendible.

Recap: Symmetric subspace projector

Normalized measure on pure states

Denote by $\mathcal{D}_1(\mathbb{C}^d)$ the set of pure states on $\mathbb{C}^d$. Parametrizing $|\phi\rangle = U|\phi_0\rangle$ for some fixed state $|\phi_0\rangle$ and unitary $U \in \mathcal{U}_d$, the Haar measure on $\mathcal{U}^d$ induces a normalized measure $\mathrm{d}\phi$ on $\mathcal{D}_1(\mathbb{C}^d)$ via

$$\int_{\mathcal{D}_1(\mathbb{C}^d)} \mathrm{d}\phi \, f(|\phi\rangle) = \int_{\mathcal{U}_d} \mathrm{d}U \, f(U|\phi_0\rangle).$$

Formula for the symmetric subspace projector

$$\Pi_{\mathrm{sym}} = \binom{n+d-1}{n} \int_{\mathcal{D}_1(\mathbb{C}^d)} \mathrm{d}\phi \, |\phi\rangle\langle\phi|^{\otimes n}.$$