Recap: Multipartite Werner states

Definition

Let $\mathcal{H}_{A_i} = \mathbb{C}^d$ for $i = 1, \ldots, n$. A state $\rho_{A_1 \cdots A_n}$ is called a *multipartite Werner state* if

$$U_A^{\otimes n} \rho_{A_1 \cdots A_n} (U_A^\dagger)^{\otimes n} = \rho_{A_1 \cdots A_n} \qquad \text{for all } U_A \in \mathcal{U}_d.$$

Structure of multipartite Werner states

$$\rho_{A_1 \cdots A_n} = \bigoplus_{\lambda \vdash_d n} x_\lambda \rho_\lambda \otimes \frac{1}{m_{\lambda,d}} \mathbb{1}_{U_\lambda^d}$$

with prob. dist. $(x_\lambda)_{\lambda \vdash_d n}$, quantum states ρ_λ on V_λ for $\lambda \vdash_d n$, and $m_{\lambda,d} = \dim U_\lambda^d$.

Structure of *symmetric* (≡permutation-invariant) multipartite Werner states

$$\rho_{A_1 \cdots A_n} = \bigoplus_{\lambda \vdash_d n} x_\lambda \frac{1}{d_\lambda} \mathbb{1}_{V_\lambda} \otimes \frac{1}{m_{\lambda,d}} \mathbb{1}_{U_\lambda^d}.$$

Recap: Isotropic states

Definition

A state ρ_{AB} on systems AB with $\mathcal{H}_A \cong \mathcal{H}_B \cong \mathbb{C}^d$ is called *isotropic* [HH99] if

$$\left(U \otimes \overline{U}\right) \rho_{AB} \left(U \otimes \overline{U}\right)^\dagger = \rho_{AB} \quad \text{for all } U \in \mathcal{U}_d.$$

Structure of isotropic states

$$\rho_{AB} = (1 - x)|\Phi^+\rangle\langle\Phi^+|_{AB} + x \frac{1}{d^2} \mathbb{1}_{AB} \qquad \text{for } x \in \left[0, \frac{d^2}{d^2 - 1}\right].$$

Entanglement in isotropic states

(i) Let σ_{AB} be arbitrary with $\beta := \operatorname{tr}(\sigma_{AB}\Phi^+_{AB}) = \langle\Phi^+|\sigma_{AB}|\Phi^+\rangle$. Then

$$\int_{\mathcal{U}_d} (U \otimes \overline{U})\sigma_{AB}(U \otimes \overline{U})^\dagger = \rho_{AB}(y) \qquad \text{with } y = \frac{d^2}{d^2 - 1}(1 - \beta).$$

(ii) ρ_{AB} is separable iff $x \geq \frac{d}{d+1}$.

Comparing Werner and isotropic states

For 2 qubits, Werner and isotropic states have an equivalent entanglement structure (see exercises):

Two-qubit Werner and isotropic states

Any Werner state on $\mathbb{C}^2 \otimes \mathbb{C}^2$ is local unitary equivalent to an isotropic state.

However, for local dimension $d \geq 3$ the two families of states are not even equivalent via a global unitary except for the completely mixed state $\frac{1}{d^2}\mathbb{1}_{AB}$, which obviously has both $(U \otimes U)$ and $(U \otimes \overline{U})$-symmetry.