End-of-term presentations

Timeline

- **Thursday, Oct 16:** Choose group and topic
  (from list posted below, on course website, and on Campuswire).
- **Thursday, Nov 13:** Submit a 1-page talk outline to me.
- **December:** Presentations (exact dates TBD, but prepare to be ready by Dec 2).
  10-15min speaking time per group member.

List of topics for end-of-term presentations

- Character theory, Schur polynomials, symmetric functions
- Testing separability
- NPT bound entanglement and Werner states
- Unitary designs
- Pseudorandomness and pseudoentanglement
- Post-selection technique and applications in QKD and channel coding
- Weingarten calculus and free probability
- More De Finetti theorems
- Commutant of the Clifford group
- Studying entanglement in tripartite systems using Schur-Weyl duality
- Consensus for Quantum Networks: Symmetry From Gossip Interactions (already assigned)

Pointers to literature will be provided.

Announcement

Thursday, Oct 9

Interactive problem session led by Jacob and Theshani (instead of class, same time and same place).

Exercises to be discussed: 3.1, 3.5, 3.6, 3.9, and 3.12.

Typed-up solutions will be provided afterwards.

Recap: Duality theorem

Duality theorem

Let (φ, V) be a representation of a finite group G with isotypical decomposition

$$V = \bigoplus_\alpha V_\alpha \otimes \mathbb{C}^{n_\alpha}$$

into pairwise inequivalent irreducible representations $(\varphi_\alpha, V_\alpha)$ with multiplicity n_α. Let $\mathcal{A} \subset \mathrm{End}(V)$ be the subalgebra generated by φ, and set $\mathcal{B} = \mathcal{A}'$. Then:

(i) $\mathcal{A} \cong \bigoplus_\alpha \mathrm{End}(V_\alpha) \otimes \mathbb{1}_{\mathbb{C}^{n_\alpha}}$

(ii) $\mathcal{B} \cong \bigoplus_\alpha \mathbb{1}_{V_\alpha} \otimes \mathrm{End}(\mathbb{C}^{n_\alpha})$

(iii) $\mathcal{B}' = (\mathcal{A}')' = \mathcal{A}$

The symmetric group has a representation on $(\mathbb{C}^d)^{\otimes n}$ by permuting tensor factors:

$$\varphi(\pi)(|\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle) = |\psi_{\pi^{-1}(1)}\rangle \otimes \cdots \otimes |\psi_{\pi^{-1}(n)}\rangle.$$

The unitary group also has a representation on $(\mathbb{C}^d)^{\otimes n}$ by acting diagonally:

$$\omega(U)(|\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle) = U|\psi_1\rangle \otimes \cdots \otimes U|\psi_n\rangle.$$

Recap: Symmetric subspace

Definition

The *symmetric subspace* $\mathrm{Sym}^n(V)$, also called the *n-th symmetric power* of V, is the subspace invariant under the action of S_n:

$$\mathrm{Sym}^n(V) = (V^{\otimes n})^{S_n} = \left\{ |v\rangle \in V^{\otimes n} \,:\, \varphi(\pi)|v\rangle = |v\rangle \text{ for all } \pi \in S_n \right\}.$$

Generating set for the symmetric subspace

$$\mathrm{Sym}^n(V) = \mathrm{span}\{|v\rangle^{\otimes n} \,:\, |v\rangle \in V\}.$$