Announcements

Visitor: Daniel Spiegel (Harvard)

Visiting all week.

Seminar talk on Wed, Oct 22, 4-5pm in 1022 Lincoln Hall:

"A Classifying Space for Phases of Matrix Product States"

Recap: Young symmetrizer

Let T be a standard Young tableau of shape $\lambda \vdash_d n$. Define two subgroups R_T, C_T of S_n as

$$R_T \coloneqq \{\pi \in S_n : \pi \text{ permutes integers within rows of } T\}$$
$$C_T \coloneqq \{\pi \in S_n : \pi \text{ permutes integers within columns of } T\}.$$

We define two elements in $\mathbb{C}[S_n]$:

$$r_T \coloneqq \sum_{\pi \in R_T} \pi \qquad c_T \coloneqq \sum_{\pi \in C_T} \operatorname{sgn}(\pi)\pi.$$

Young symmetrizer

For a given standard Young tableau T of shape $\lambda \vdash n$, the *Young symmetrizer* e_T is defined as $e_T \coloneqq r_T c_T$.

The projection onto the irreducible representation V_λ of S_n in $(\mathbb{C}^d)^{\otimes n}$ is equal to

$$f_T \coloneqq \frac{d_\lambda}{n!} e_T.$$

Recap: Irreps of S_n and $\mathcal{U}_d$

Irreps of S_n and $\mathcal{U}_d$

Let $d = \dim V$ and $|v\rangle \in V^{\otimes n}$ be non-zero. For a standard Young tableau T of shape $\lambda \vdash n$, consider the Young symmetrizer e_T. Let p be the number of parts of the partition λ (or the number of non-zero rows of the Young diagram λ).

- If $p \le d$, then $\mathbb{C}[S_n]e_T|v\rangle$ is an irreducible representation of S_n isomorphic to the Specht module V_λ.
- If $p \le d$, then $e_T V^{\otimes n}$ is an irreducible representation of $\mathrm{GL}(V)$ (or $\mathcal{U}_d$) on $V^{\otimes n}$. These are inequivalent for Young tableaux of different shape.
- Using the above, we have the Schur-Weyl decomposition of $V^{\otimes n}$ with $d = \dim V$ as an $S_n \times \mathcal{U}_d$ representation:

$$V^{\otimes n} = \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$