Recap: Approximate cloning

Setup

Given: d-dim. Hilbert space $\mathcal{H}$ and N copies of an unknown state $|\psi\rangle \in \mathcal{H}$.

Goal: Produce an approximation of M copies of ψ for some $M > N$.

Figure of merit: Let T be the approximate cloning map, $T : \mathcal{L}(\mathcal{H}^{\otimes N}) \to \mathcal{L}(\mathcal{H}^{\otimes M})$.

We define the *worst case fidelity*

$$F(T) = \inf_{|\psi\rangle} F\left(\psi^{\otimes M}, T(\psi^{\otimes N})\right)^2 = \inf_{|\psi\rangle} \mathrm{tr}(\psi^{\otimes M} T(\psi^{\otimes n})),$$

where we used the fact that $F(|\phi\rangle\langle\phi|, \rho)^2 = \langle\phi|\rho|\phi\rangle = \mathrm{tr}\,(|\phi\rangle\langle\phi|\rho)$.

Bound on worst-case fidelity [Wer98]

Define $d_N := \dim \mathrm{Sym}^N(\mathcal{H}) = \binom{d+N-1}{N}$. For any cloning map $T : \mathcal{L}(\mathcal{H}^{\otimes N}) \to \mathcal{L}(\mathcal{H}^{\otimes M})$,

$$F(T) \leq \frac{d_N}{d_M} = \binom{d+N-1}{N}\binom{d+M-1}{M}^{-1}.$$

Recap: Optimal approximate cloning

The fidelity bound in the lemma is achieved by the following map:

$$T_{\mathrm{opt}}(X) = \frac{d_N}{d_M} \Pi_M \left(X \otimes \mathbb{1}_d^{\otimes M-N}\right) \Pi_M.$$

The action of this map on $X \in \mathcal{L}(\mathcal{H}^{\otimes N})$ consists of the following three steps:

Step 1. Extend state trivially from $\mathcal{H}^{\otimes N}$ to $\mathcal{H}^{\otimes M}$.
Step 2. Project down to symmetric subspace $\mathrm{Sym}^M(\mathcal{H})$.
Step 3. Normalize to get a quantum state.

T_{opt} achieves the fidelity bound [Wer98]

$$F(T_{\mathrm{opt}}) = \binom{d+N-1}{N}\binom{d+M-1}{M}^{-1} \geq 1 - \frac{Kd}{N}$$

This bound shows that, for $N, M \to \infty$ with $K = M - N$ fixed, approximate cloning becomes possible with the worst-case fidelity $F(T)$ arbitrarily close to 1.

End-of-term presentations

Timeline

- **Thursday, Nov 13:** Every group submits a 1-page talk outline to me.
- **December 2–??:** Presentations.

Tentative schedule:

- Two groups each on Dec 2, Dec 4, Dec 9 (last regular class).
- Remaining presentations on Dec 11 in the morning??