End-of-term presentations

Timeline

- **Thursday, Oct 16:** Choose group and topic
  (from list posted below, on course website, and on Campuswire).
- **Thursday, Nov 13:** Submit a 1-page talk outline to me.
- **December:** Presentations (exact dates TBD, but prepare to be ready by Dec 2).
  10-15min speaking time per group member.

Recap: De Finetti theorem for permutation-invariant states

Lemma

Let $\mathcal{H}_{A_i} = \mathbb{C}^d$ for $i = 1, \ldots, n$ and $\rho_{A_1 \cdots A_n}$ be permutation invariant. Then $\rho_{A_1 \cdots A_n}$ has a purification $|\psi^\rho\rangle \in \mathrm{Sym}^n(\mathbb{C}^d \otimes \mathbb{C}^d)$.

De Finetti theorem

Let $\mathcal{H}_{A_i} = \mathbb{C}^d$ for $i = 1, \ldots, n$ and $\rho_{A_1 \cdots A_n}$ be permutation-invariant. For any $k < n$,

$$D\left(\rho_{A_1 \cdots A_k}, \int \mathrm{d}\mu_\rho(\sigma)\, \sigma_A^{\otimes k}\right) \leq \sqrt{\frac{d^2 k}{n-k}},$$

where $\mathrm{d}\mu_\rho(\sigma)$ is a measure on the space of mixed states on $\mathbb{C}^d$ that depends on ρ.

Recap: No-cloning and approximate cloning

No-cloning theorem

Let A, B be d-dimensional quantum systems. There is no unitary $U \in \mathcal{U}_d$ that achieves the transformation

$$U: |\psi\rangle_A \otimes |0\rangle_B \mapsto |\psi\rangle_A \otimes |\psi\rangle_B$$

for arbitrary $|\psi\rangle \in \mathcal{H}_{\mathcal{A}}$. Here $|0\rangle_B$ is some reference state.

Approximate cloning

Given: d-dim. Hilbert space $\mathcal{H}$ and N copies of an unknown state $|\psi\rangle \in \mathcal{H}$.

Goal: Produce an approximation of $\psi^{\otimes M}$ for some $M > N$.

Figure of merit: Let T be the approximate cloning map, $T: \mathcal{L}(\mathcal{H}^{\otimes N}) \rightarrow \mathcal{L}(\mathcal{H}^{\otimes M})$.

We define the *worst case fidelity*

$$F(T) = \inf_{|\psi\rangle} F\left(\psi^{\otimes M}, T(\psi^{\otimes N})\right)^2 = \inf_{|\psi\rangle} \mathrm{tr}(\psi^{\otimes M} T(\psi^{\otimes n})).$$