Recap: Weak Schur sampling

Weak Schur sampling

Let ρ be a density operator with spectrum $r = (r_1, \ldots, r_d)$ where $r_1 \geq r_2 \geq \cdots \geq r_d \geq 0$, let $\lambda = (\lambda_1, \ldots, \lambda_d) \vdash_d n$ and set $\bar{\lambda} = \frac{\lambda}{n}$. Then,

$$\mathrm{tr}(P_\lambda \rho^{\otimes n}) \leq (n+1)^{\frac{d(d-1)}{2}} \exp\left(-nD(\bar{\lambda} \| r)\right).$$

Extend this bound to a set S of possible spectra by defining

$$P_S = \sum_{\substack{\lambda \vdash n, \\ \bar{\lambda} \in S}} P_\lambda.$$

Probability bound

$$\mathrm{tr}(P_S \rho^{\otimes n}) \leq (n+1)^{\frac{d(d-1)}{2}} \exp\left(-n \min\left\{D(\bar{\lambda} \| r) : \lambda \vdash n, \bar{\lambda} \in S\right\}\right)$$

Recap: Asymptotics of spectrum estimation

Asymptotic spectrum estimation

Let ρ be a quantum state with (ordered) spectrum $r = (r_1, ..., r_d)$, and for given $\varepsilon > 0$ let

$$P_X = \sum_{\substack{\lambda \vdash n \\ \bar{\lambda} \in B_\varepsilon(r)}} P_\lambda.$$

Then for any $\delta > 0$ there exists n_0 such that for all $n \geq n_0$ we have $\mathrm{tr}(P_X \rho^{\otimes n}) \geq 1 - \delta$.

Sample complexity of spectrum estimation; [OW15]

For a mixed state ρ with ordered spectrum $r = (r_1, \ldots, r_d)$, we have for any $\varepsilon > 0$ that

$$\Pr\left[d_{\mathrm{TV}}(\bar{\lambda}, r) > \varepsilon\right] \leq (n+1)^{d(d+1)/2} \exp(-2n\varepsilon^2).$$

Hence, $O(d^2/\varepsilon^2)\log(d/\varepsilon)\log(1/\delta)$ samples are sufficient to output an estimate $\bar{\lambda}$ for the spectrum of ρ satisfying $d_{\mathrm{TV}}(\bar{\lambda}, r) \leq \varepsilon$ with probability at least $1 - \delta$.