Recap: Majorization in Schur-Weyl duality

Goal: bound the probability $\mathrm{tr}(P_\lambda \rho^{\otimes n})$ of obtaining outcome "λ" in weak Schur sampling.

We will use the concept of *majorization* [MOA11] for this.

Majorization

Let $x, y \in \mathbb{R}^d$, and denote by $x^\downarrow$, $y^\downarrow$ the vectors of components of x, y sorted in non-increasing order (e.g. $x_1^\downarrow \geq \cdots \geq x_d^\downarrow$). Then y is said to *majorize* x, in symbols $x \prec y$ if

$$\sum_{i=1}^{q} x_i^\downarrow \leq \sum_{i=1}^{q} y_i^\downarrow \text{ for all } q = 1, \ldots, d-1, \text{ and } \sum_{i=1}^{d} x_i = \sum_{i=1}^{d} y_i.$$

Lemma

Let $|v\rangle \in B$ with frequency distribution f, and let T be the standard Young tableau of shape $\lambda \vdash_d n$. Then $e_T |v\rangle = 0$ unless $f \prec \lambda$.

Recap: Bounding probabilities in weak Schur sampling

Relative entropy (also called *Kullback-Leibler divergence*) between two probability distributions p, q over the same alphabet:

$$D(p\|q) = \begin{cases} \sum_i p_i \log \frac{p_i}{q_i} & \text{if } \operatorname{supp}(p) := \{i : p_i \neq 0\} \subseteq \operatorname{supp}(q); \\ \infty & \text{otherwise.} \end{cases}$$

"Distance measure" on probability distributions: $D(p\|q) \geq 0$ and $D(p\|q) = 0$ iff $p = q$.

Probability of obtaining outcome "λ" in weak Schur sampling is exponentially small if $\overline{\lambda}$ is far from the spectrum of ρ in relative entropy distance:

Weak Schur sampling

Let ρ be a density operator with spectrum $r = (r_1, \ldots, r_d)$ where $r_1 \geq r_2 \geq \cdots \geq r_d \geq 0$, let $\lambda = (\lambda_1, \ldots, \lambda_d) \vdash_d n$ and set $\overline{\lambda} = \frac{\lambda}{n}$. Then,

$$\operatorname{tr}(P_\lambda \rho^{\otimes n}) \leq (n+1)^{\frac{d(d-1)}{2}} \exp\left(-nD\left(\overline{\lambda}\|r\right)\right).$$

End-of-term presentations

Timeline

- **Thursday, Nov 13:** Every group submits a 1-page talk outline to me.
- **December 2–11:** Presentations.

Tentative schedule:

- Two groups each on Dec 2, Dec 4, Dec 9 (last regular class).
- Remaining presentations on **Dec 11, 9am to 12pm, in 1027 Lincoln Hall**.