

Recap: Approximate cloning

We showed optimality of the cloning map

$$T_{\text{opt}}(X) = \frac{d_N}{d_M} \Pi_M (X \otimes \mathbb{1}_d^{\otimes M-N}) \Pi_M$$

for the **worst-case fidelity**, achieving $F(T_{\text{opt}}) = \binom{d+N-1}{N} \binom{d+M-1}{M}^{-1} \geq 1 - \frac{d(M-N)}{N}$.

In fact, this map is also optimal for the weaker **average fidelity**

$$F_{\text{avg}}(T) = \int d\psi F(\psi^{\otimes n}, T(\psi^{\otimes N}))^2.$$

There are many more settings for approximate cloning:
single-copy error, state-dependent cloning, asymmetric cloning, ...

Recap: Spectrum estimation

In this section we are interested in the task of estimating the spectrum (i.e., the set of eigenvalues) of an unknown density operator ρ of a quantum system.

Assumptions

- (i) We have access to an experiment that prepares the system (exactly) in the state ρ .
- (ii) We can run this experiment n times and perform joint measurements on all n copies at the same time. The goal is to estimate the spectrum of ρ by making a suitable measurement on $\rho^{\otimes n}$.

The goal is to devise a strategy that correctly estimates the true spectrum of ρ with probability approaching 1 as $n \rightarrow \infty$.

Recap: Symmetries of spectrum estimation

Permutation invariance

Source state $\rho^{\otimes n}$ is permutation-invariant, so measurement operators M_i can be chosen permutation-invariant w.l.o.g. as well.

Unitary invariance

Spectrum of a state is a unitarily invariant object, so measurement operators M_i can be chosen to be $U^{\otimes n}$ -invariant by a result of [MW16].

Weak Schur sampling

Let P_λ be the isotypical projector onto the λ -summand in the Schur-Weyl decomp.

$$(\mathbb{C}^d)^{\otimes n} = \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$

$\{P_\lambda\}_{\lambda \vdash_d n}$ is a measurement whose effect operators are permutation- and $U^{\otimes n}$ -invariant.

Outcome “ λ ” \rightarrow Spectrum estimator $\bar{\lambda} = \frac{1}{n}\lambda$.

End-of-term presentations

Timeline

- **Thursday, Nov 13:** Every group submits a 1-page talk outline to me.
- **December 2–11:** Presentations.

Tentative schedule:

- Two groups each on Dec 2, Dec 4, Dec 9 (last regular class).
- Remaining presentations on Dec 11, 9am to 12pm, **classroom TBD.**