Math 595 Representation-theoretic methods in QIT

Irreducible representations of the symmetric and unitary groups

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Conjugacy classes of the symmetric group

Conjugacy classes and irreps

Recall from character theory that for a finite group G, the number of irreducible representations of G is equal to the number of conjugacy classes of G.

Conjugacy classes form a partition of a finite group: The relation $\sim$ on G defined as $g \sim h$ if and only if there exists $s \in G$ such that $g = shs^{-1}$ is an equivalence relation.

The *conjugacy classes* $C_1, \ldots, C_k$ of G are the equivalence classes with respect to this relation, and hence form a partition of G.

We will need the following facts about permutations (see, e.g., [Goo14]):

Facts about permutations

(i) Every permutation $\pi \in S_n$ can be written uniquely as a product of disjoint cycles, e.g., $\pi = (13)(2)(465) \in S_6$. The *cycle type* of a permutation $\pi \in S_n$ is the tuple of cycle lengths in non-increasing order. For example, $\pi = (14)(236)(58)(7)$ has cycle type $(3, 2, 2, 1)$.

Conjugacy classes of the symmetric group

Facts about permutations

(ii) Cycle types $(\lambda_1, \ldots, \lambda_d)$ of a permutation $\pi \in S_n$ form an ordered partition of n, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d \geq 0$ and $\sum_{i=1}^{d} \lambda_i = n$. We use the notation $\lambda \vdash_d n$ for an ordered partition of n into at most d parts. Note: If $d < n$ then not all possible partitions or cycle types appear.

(iii) Two permutations $\pi, \pi' \in S_n$ are conjugate iff they have the same cycle type.

To see (iii), let $(i_1, \ldots, i_k)$ be a cycle of length $k \leq n$ and $\sigma \in S_n$ be arbitrary. Then,

$$\sigma(i_1, \ldots, i_k)\sigma^{-1} = (\sigma(i_1), \ldots, \sigma(i_k)).$$

It follows from (i)-(iii) above that the conjugacy classes of S_n, and hence its irreducible representations, are indexed by the ordered partitions of n into n parts.

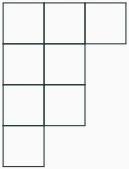
Young diagrams and Young tableaux

Partitions as diagrams

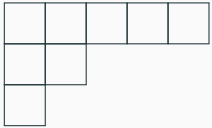
Definition

Let $\lambda = (\lambda_1, \dots, \lambda_d) \vdash_d n$ be a partition of n into at most d parts. The *Young diagram* corresponding to $\lambda \vdash_d n$ is an arrangement of n boxes into d rows such that the i-th row has length λ_i.

Examples: two Young diagrams $\lambda_1, \lambda_2 \vdash_4 8$



$\lambda_1 = (3, 2, 2, 1) \vdash_4 8$



$\lambda_2 = (5, 2, 1, 0) \vdash_4 8.$

Young tableaux

Definition (Young tableaux)

- A *Young tableau* is a Young diagram λ with boxes labeled with numbers $\{1, \ldots, N\}$ (we can have $N \neq n$). We call λ the *shape* of the Young tableau T.
- A *standard Young tableau* is a Young tableau with $N = n$, and the labels are strictly increasing along rows (left to right) and along columns (top to bottom).
- A *semistandard Young tableau* is a Young tableau whose labels are non-decreasing along rows and strictly increasing along columns.

The standard Young tableaux of shape $\lambda = (3, 2)$ are

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 \\ \cline{1-2}\end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & 5 \\ \cline{1-2}\end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 3 & 4 \\ \cline{1-2}\end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & 5 \\ \cline{1-2}\end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline 2 & 4 \\ \cline{1-2}\end{array}.$$

The semistandard Young tableaux of shape $\lambda = (3, 2)$ with numbering $\{1, 2\}$ are

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 2 & 2 \\ \cline{1-2}\end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline 2 & 2 \\ \cline{1-2}\end{array}.$$

Indexing irreps in Schur-Weyl decomposition

Schur-Weyl duality gives a decomposition

$$\left(\mathbb{C}^d\right)^{\otimes n} = \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$

Irreps in the Schur-Weyl decomposition

In the above decomposition,

- the index $\lambda \vdash_d n$ runs over the partitions of n into at most d parts, or alternatively the set of Young diagrams of n boxes and at most d rows;
- the irrep V_λ of S_n has an orthonormal basis indexed by the set of **standard Young tableaux** of shape $\lambda \vdash_d n$.
- the irrep U_λ^d of $\mathcal{U}_d$ has an orthonormal basis indexed by the set of **semistandard Young tableaux** of shape $\lambda \vdash_d n$ and numbering $\{1, \ldots, d\}$.

Combinatorial formulas for irrep dimensions

Let $d, n \in \mathbb{N}$. The number of standard Young tableaux of shape $\lambda \vdash_d n$ is equal to

$$d_\lambda := \frac{n!}{\prod_{(i,j)\in\lambda} h(i,j)},$$

where for a box (i,j) in row i and column j of λ we define the **hook length**

$$h(i,j) = \#\{\text{boxes to the right of } (i,j)\} + \#\{\text{boxes below } (i,j)\} + \text{the box } (i,j) \text{ itself.}$$

Example

Let $\lambda = (4,2,1)$ be the Young diagram $\begin{array}{|c|c|c|c|}\hline 6 & 4 & 2 & 1 \\ \hline 3 & 1 \\ \cline{1-2} 1 \\ \cline{1-1}\end{array}$, where we filled each box (i,j) with its hook length (i,j). Then,

$$d_\lambda = \frac{(4+2+1)!}{6\cdot 4\cdot 2\cdot 1\cdot 3\cdot 1\cdot 1} = \frac{7!}{6\cdot 4\cdot 3\cdot 2} = 7\cdot 5 = 35.$$

Combinatorial formulas for irrep dimensions

The number of semistandard Young tableaux of shape $\lambda \vdash_d n$ is equal to

$$m_{\lambda,d} = \prod_{1 \le i < j \le d} \frac{\lambda_i - \lambda_j + j - i}{j - i}.$$

Example: $d = 3$

The number of semistandard Young tableaux of shape $\lambda = (4, 2, 1)$ with numbering $\{1, 2, 3\}$ is

$$m_{\lambda,3} = \frac{4 - 2 + 2 - 1}{2 - 1} \cdot \frac{4 - 1 + 3 - 1}{3 - 1} \cdot \frac{2 - 1 + 3 - 2}{3 - 2} = \frac{3}{1} \cdot \frac{5}{2} \cdot \frac{2}{1} = 15.$$

Example: $d = 4$

The number of semistandard Young tableaux of shape $\lambda = (4, 2, 1)$ with numbering $\{1, 2, 3\}$ is

$$m_{\lambda,4} = \frac{4 - 2 + 1}{1} \cdot \frac{4 - 1 + 2}{2} \cdot \frac{4 - 0 + 3}{3} \cdot \frac{2 - 1 + 1}{1} \cdot \frac{2 - 0 + 2}{2} \cdot \frac{1 - 0 + 1}{1} = 3 \cdot \frac{5}{2} \cdot \frac{7}{3} \cdot 2 \cdot 2 \cdot 2 = 140.$$

Combinatorial formulas for irrep dimensions

The 15 semistandard Young tableaux of shape $\lambda = (4, 2, 1)$ with numbering $\{1, 2, 3\}$ are

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&1\\\hline 2&2\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&2&2\\\hline 2&2\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&2&3\\\hline 2&2\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&3&3\\\hline 2&2\\\cline{1-2} 3\\\cline{1-1}\end{array}$$

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&1\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&2&2\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&2&3\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&1&3&3\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array}\ .$$

$$\begin{array}{|c|c|c|c|}\hline 1&2&2&2\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&2&2&3\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array},\quad \begin{array}{|c|c|c|c|}\hline 1&2&3&3\\\hline 2&3\\\cline{1-2} 3\\\cline{1-1}\end{array}$$

Constructing the irreps of the symmetric group

Row and column group of a Young tableau

Let T be a standard Young tableau of shape $\lambda \vdash_d n$. Define two subgroups R_T, C_T of S_n as

$$R_T \coloneqq \{\pi \in S_n : \pi \text{ permutes integers within rows of } T\}$$
$$C_T \coloneqq \{\pi \in S_n : \pi \text{ permutes integers within columns of } T\}.$$

Example

The table below lists standard Young tableau T of shape $\lambda \vdash_3 6$ and the corresponding groups R_T, C_T.

T	$\begin{array}{\|c\|c\|c\|}\hline 1 & 2 & 3 \\ \hline \end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} \end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array}$	$\begin{array}{\|c\|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}$
R_T	S_3	$\{e, (12)(3)\} \cong S_2$	$\{e, (13)(2)\} \cong S_2$	$\{e\} \cong S_1$
C_T	$\{e\} \cong S_1$	$\{e, (13)(2)\} \cong S_2$	$\{e, (12)(3)\} \cong S_2$	S_3

Young symmetrizer

We define two elements in $\mathbb{C}[S_n]$:

$$r_T := \sum_{\pi \in R_T} \pi \qquad\qquad c_T := \sum_{\pi \in C_T} \operatorname{sgn}(\pi)\pi.$$

Young symmetrizer

For a given standard Young tableau T of shape $\lambda \vdash n$, the *Young symmetrizer* e_T is defined as $e_T := r_T c_T$.

Example

(i) Let $\lambda = (n) \vdash n$ and $T = \boxed{1}\boxed{\dots}\boxed{n}$. Then $c_T = \{e\}$, $R_T = S_n$ and $e_T = \sum_{\pi \in S_n} \pi$.

(ii) Let $\lambda = (1, \dots, 1)$ and $T = \begin{array}{|c|}\hline 1 \\ \hline \vdots \\ \hline n \\ \hline\end{array}$. Then $e_T = \sum_{\pi \in S_n} \operatorname{sgn}(\pi)\pi$.

Irreps of S_n

Specht modules

Let T be a Young tableau of shape $\lambda \vdash n$, and let e_T be the corresponding Young symmetrizer. Then $f_T := \frac{d_\lambda}{n!} e_T$ is the minimal projection in $\mathbb{C}[S_n]$ corresponding to the irreducible representation V_λ of S_n, that is, $V_\lambda \cong \mathbb{C}[S_n]e_T$. The V_λ are called *Specht modules*. Every irreducible representation of S_n is isomorphic to a Specht module V_λ for some $\lambda \vdash n$, and $V_\lambda \not\cong V_{\lambda'}$ for $\lambda \neq \lambda'$. (Proof: [Chr06] or [Alc18])

Denote by $\mathrm{SYT}(\lambda)$ the set of all standard Young tableaux of shape λ, and let $T \in \mathrm{SYT}(\lambda)$. Then the concrete realization of the group algebra element f_T on $(\mathbb{C}^d)^{\otimes n}$ is

$$\Pi_T = \frac{d_\lambda}{n!} \sum_{\pi \in R_T, \sigma \in C_T} \mathrm{sgn}(\sigma)\varphi(\pi)\varphi(\sigma).$$

Recall the isotypical projection Π_λ onto the isotypical component $V_\lambda \otimes U_\lambda^d$ corresponding to the irrep V_λ of S_n. Then we have the relation

$$\Pi_\lambda = \sum_{T \in \mathrm{SYT}(\lambda)} \Pi_T.$$

Constructing the irreps of the unitary group

Irreps of $\mathcal{U}_d$

Recall the Schur-Weyl decomposition

$$(\mathbb{C}^d)^{\otimes n} \cong \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$

We know that the dimension of the S_n-irrep V_λ is equal to the number d_λ of standard Young tableaux, which can be computed using the hook length formula.

Irreps of $\mathcal{U}_d$

Let $\lambda \vdash_d n$ be a Young diagram. Then for each standard Young tableau T of shape λ, the subspace $e_T(\mathbb{C}^d)^{\otimes n}$ is an irreducible representation of $\mathcal{U}_d$ (or equivalently $\mathrm{GL}(\mathbb{C}^d)$) isomorphic to U_λ^d.

For an easy proof see [Alc18, Prop. 6.2].

Constructing a basis for unitary group irreps

Let $\{|i\rangle\}_{i=1}^{d}$ be the standard basis of $\mathbb{C}^d$. Consider a tensor product basis of $(\mathbb{C}^d)^{\otimes n}$:

$$\mathcal{B} = \{|i_1\rangle \otimes \ldots \otimes |i_n\rangle : i_j \in [d]\}$$

For a fixed Young tableau T of shape λ and basis vector $|v\rangle = |i_1\rangle \otimes \ldots \otimes |i_n\rangle \in \mathcal{B}$, construct a Young tableau $T_{|v\rangle}$ obtained from replacing j in T with the label i_j.

Example for $d = 2$ and $n = 4$

$$|v\rangle = |1\rangle \otimes |2\rangle \otimes |1\rangle \otimes |1\rangle : \quad S = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} \longrightarrow S_{|v\rangle} = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 1 \\ \cline{1-1}\end{array}$$

$$T = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} \longrightarrow T_{|v\rangle} = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array}.$$

Note that $T_{|v\rangle}$ is a semistandard Young tableau, while $S_{|v\rangle}$ is not.

Constructing a basis for unitary group irreps

Example for $d = 2$ and $n = 4$

$$|v\rangle = |1\rangle \otimes |2\rangle \otimes |1\rangle \otimes |1\rangle : \quad S = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} \longrightarrow S_{|v\rangle} = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 1 \\ \cline{1-1}\end{array}$$

$$T = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} \longrightarrow T_{|v\rangle} = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array}.$$

Observation: The Young symmetrizer e_S annihilates the basis vector $|v\rangle$, since there is a repetition in $S_{|v\rangle}$ in the first column, which is *antisymmetrized* by e_S.

Corollary

If T is a Young tableau with more than d rows, then any basis vector $|v\rangle \in \mathcal{B}$ is annihilated by e_T because of the antisymmetrization along columns.

This is why only irreps corresponding to Young diagrams with at most d rows appear in the Schur-Weyl decomposition.

Constructing a basis for unitary group irreps

Generalization:

(i) For given $|v\rangle \in \mathcal{B}$ let $\mu = (\mu_i)_{i=1}^d$ be the vector where μ_i is the number of times the basis vector $|i\rangle$ appears in $|v\rangle$.

(ii) Denote by $\mu^\downarrow$ the vector obtained from μ by sorting its components in non-increasing order. If the standard Young tableau T has shape λ, then e_T annihilates $|v\rangle$ whenever

$$\sum_{i=1}^{k} \mu_i^\downarrow > \sum_{i=1}^{k} \lambda_i \quad \text{for some } 1 \leq k \leq d-1.$$

(iii) This condition is the negation of the **majorization** relation, defined as $\mu^\downarrow < \lambda$ iff $\sum_{i=1}^k \mu_i^\downarrow \leq \sum_{i=1}^k \lambda_i$ for all $k = 1, \dots, d$.

All basis vectors $|v\rangle \in \mathcal{B}$ that are not annihilated by e_T span the irrep U_λ^d, and the ones corresponding to semistandard Young tableaux $T_{|v\rangle}$ form a basis.

For a concrete way of realizing the irreducible representation of a unitary U on U_λ^d, we refer to [Mol06] (for an approach using Lie theory) or [FH13, Ex. 15.57] and [Mul07, Sec. 2.1.1] (for a more direct approach).

Summary

Irreps of S_n and $\mathcal{U}_d$

Let $d = \dim V$ and $|v\rangle \in V^{\otimes n}$ be non-zero. For a standard Young tableau T of shape $\lambda \vdash n$, consider the Young symmetrizer e_T. Let p be the number of parts of the partition λ (or the number of non-zero rows of the Young diagram λ).

- If $p \leq d$, then $\mathbb{C}[S_n]e_T|v\rangle$ is an irreducible representation of S_n isomorphic to the Specht module V_λ.
- If $p \leq d$, then $e_T V^{\otimes n}$ is an irreducible representation of $\mathrm{GL}(V)$ (or $\mathcal{U}_d$) on $V^{\otimes n}$. These are inequivalent for Young tableaux of different shape.
- Using the above, we have the Schur-Weyl decomposition of $V^{\otimes n}$ with $d = \dim V$ as an $S_n \times \mathcal{U}_d$ representation:

$$V^{\otimes n} = \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$

Note that item (iii) of the proposition says that

$$d^n = \sum_{\lambda \vdash_d n} d_\lambda m_{\lambda,d}.,$$

Quantum method of types

Number of Young diagrams

Fix $n, d \in \mathbb{N}$ and consider the Schur-Weyl decomposition

$$(\mathbb{C}^d)^{\otimes n} = \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$

Simple counting argument:

Ignoring the constraint of non-decreasing row length ($\lambda_i \geq \lambda_j$ for $i \leq j$), we can put between 0 and n boxes in the first row, then in the second row, and so forth, giving the simple (over-)estimate

$$\left|\{\lambda \vdash_d n\}\right| \leq (n+1)^d.$$

$\Rightarrow$ only polynomially many terms in Schur-Weyl decomposition.

Dimension bounds for irreps

Fix $n, d \in \mathbb{N}$ and consider the Schur-Weyl decomposition

$$(\mathbb{C}^d)^{\otimes n} = \bigoplus_{\lambda \vdash_d n} V_\lambda \otimes U_\lambda^d.$$

For a given Young diagram $\lambda \vdash_d n$, define a probability distribution

$$\bar{\lambda} = (\lambda_1/n, \dots, \lambda_d/n).$$

Useful bounds on the dimension of the S_n-irrep V_λ [Har05]:

$$\exp\left(nH\left(\bar{\lambda}\right)\right)(n+d)^{-d(d+1)/2} \leq \dim V_\lambda \leq \exp\left(nH\left(\bar{\lambda}\right)\right),$$

with **Shannon entropy** $H(p) = -\sum_i p_i \log p_i$.

$\Rightarrow$ some S_n-irreps become **exponentially large** (in n).

On the other hand, all $\mathcal{U}_d$-irreps only grow **polynomially**:

$$\dim U_\lambda^d \leq (n+1)^{d(d-1)/2}.$$

Permutation-invariant states

A useful consequence of these bounds is the following (see, e.g., [BL25]):

Permutation-invariant states

Let $\mathcal{H} = (\mathbb{C}^d)^{\otimes n}$ and $\rho \in \mathcal{L}(\mathcal{H})$ be a permutation-invariant operator, $\varphi(\pi)\rho\varphi(\pi)^\dagger = \rho$ for all $\pi \in S_n$. Then ρ is determined by poly(n) parameters.

Proof.

Permutation invariance of ρ and Schur's Lemma imply that

$$\rho \cong \bigoplus_{\lambda \vdash_d n} \mathbb{1}_{V_\lambda} \otimes \rho_\lambda \qquad \text{for some } \rho_\lambda \in \mathcal{L}(U_\lambda^d).$$

Now $|\{\lambda \vdash_d n\}| \leq (n+1)^d$ and $\dim U_\lambda^d \leq (n+1)^{d(d-1)/2}$ prove the claim. □

Concentration of measure

Denote by Π_λ the isotypical projection onto the summand $V_\lambda \otimes U_\lambda^d$ in the Schur-Weyl decomposition.

Let $\rho \in \mathcal{L}(\mathbb{C}^d)$ be a quantum state of a d-dimensional system, and denote by $s = (s_1, \dots, s_d)$ the spectrum of ρ ($s_i \geq s_j$ for $i \leq j$).

Tensor copies $\rho^{\otimes n}$ are permutation-invariant: $\rho^{\otimes n} \cong \bigoplus_{\lambda \vdash_d n} \mathbb{1}_{V_\lambda} \otimes \omega_\lambda(\rho)$.

Concentration of measure

Using majorization relation and dimension bounds [CM06],

$$\begin{aligned} \exp\left(-nD\left(\bar{\lambda} \| s\right)\right)(n+d)^{-d(d+1)/2} &\leq \operatorname{tr}(\Pi_\lambda \rho^{\otimes n}) = \dim V_\lambda \operatorname{tr}[\omega_\lambda(\rho)] \\ &\leq \exp\left(-nD\left(\bar{\lambda} \| s\right)\right)(n+d)^{d(d-1)/2}, \end{aligned}$$

where $D(p\|q) = \sum_i p_i \log(p_i/q_i)$ is the relative entropy between p and q.

The weight of $\Pi_\lambda \rho^{\otimes n} \Pi_\lambda$ decays exponentially in n except for those irreps λ for which $\bar{\lambda}$ is close to the spectrum of ρ in relative entropy "distance".

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