

MATH 416 Abstract Linear Algebra

Homework Week 1 – August 26, 2021

Exercise 1 (2 points): Consider the following system of linear equations:

$$\begin{aligned}a_{11}x_1 + \cdots + a_{1n}x_n &= b_1 \\ &\vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n &= b_m,\end{aligned}$$

where $a_{ij}, b_i \in \mathbb{R}$ for $1 \leq i \leq m, 1 \leq j \leq n$. Show that the set of solutions of this system does not change under the following operations:

1. Multiply both sides of an equation by $\lambda \neq 0$.
2. Add one row to another one.

Hint: Assume there exists a solution $c = (c_1, \dots, c_n)^T$ of the original system of linear equations, and show that it is also a solution of the system transformed via the operations in 1 and 2.

Exercise 2 (10 points): Determine if the following systems of linear equations have solutions, and if yes find them. In one of the systems below there are multiple solutions. What is the dimensionality of the solution space? (1 point)

1. (3 points) System of linear equations in \mathbb{R}^3

$$\begin{aligned}x_1 + 2x_2 + 2x_3 &= 4 \\ x_1 + 3x_2 + 3x_3 &= 5 \\ 2x_1 + 6x_2 + 5x_3 &= 6\end{aligned}$$

2. (3 points) System of linear equations in \mathbb{R}^4

$$\begin{aligned}x_1 + 2x_2 + x_4 &= 7 \\ x_1 + x_2 + x_3 - x_4 &= 3 \\ 3x_1 + x_2 + 5x_3 - 7x_4 &= 1\end{aligned}$$

3. (3 points) System of linear equations in \mathbb{R}^4

$$\begin{aligned}2x_1 + x_2 + 7x_3 - 7x_4 &= 2 \\ -3x_1 + 4x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + x_2 + 4x_3 - 5x_4 &= 2\end{aligned}$$