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Enhancing classical communication networks with quantum resources

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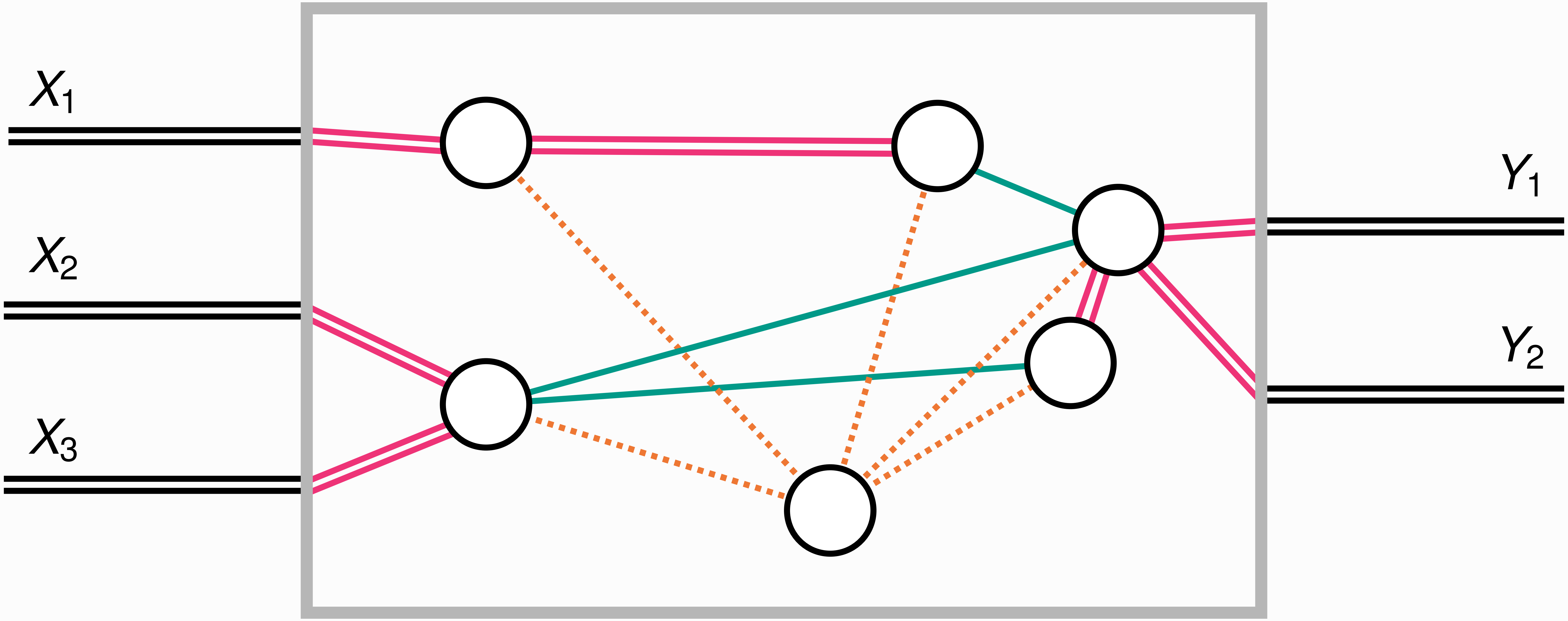
Based on:

FL, M. Alhejji, J. Levin, G. Smith, Nature Communications 11, 1497 (2020), arXiv:1909.02479
A. Seshadri, FL, V. Siddhu, G. Smith, IEEE Transactions on Inf. Th. 69.9 (2023), arXiv:2205.13538
B. Doolittle, FL, E. Chitambar, arXiv preprint (2024), arXiv:2403.02988

Introduction

How are **quantum communication networks** different from classical networks?

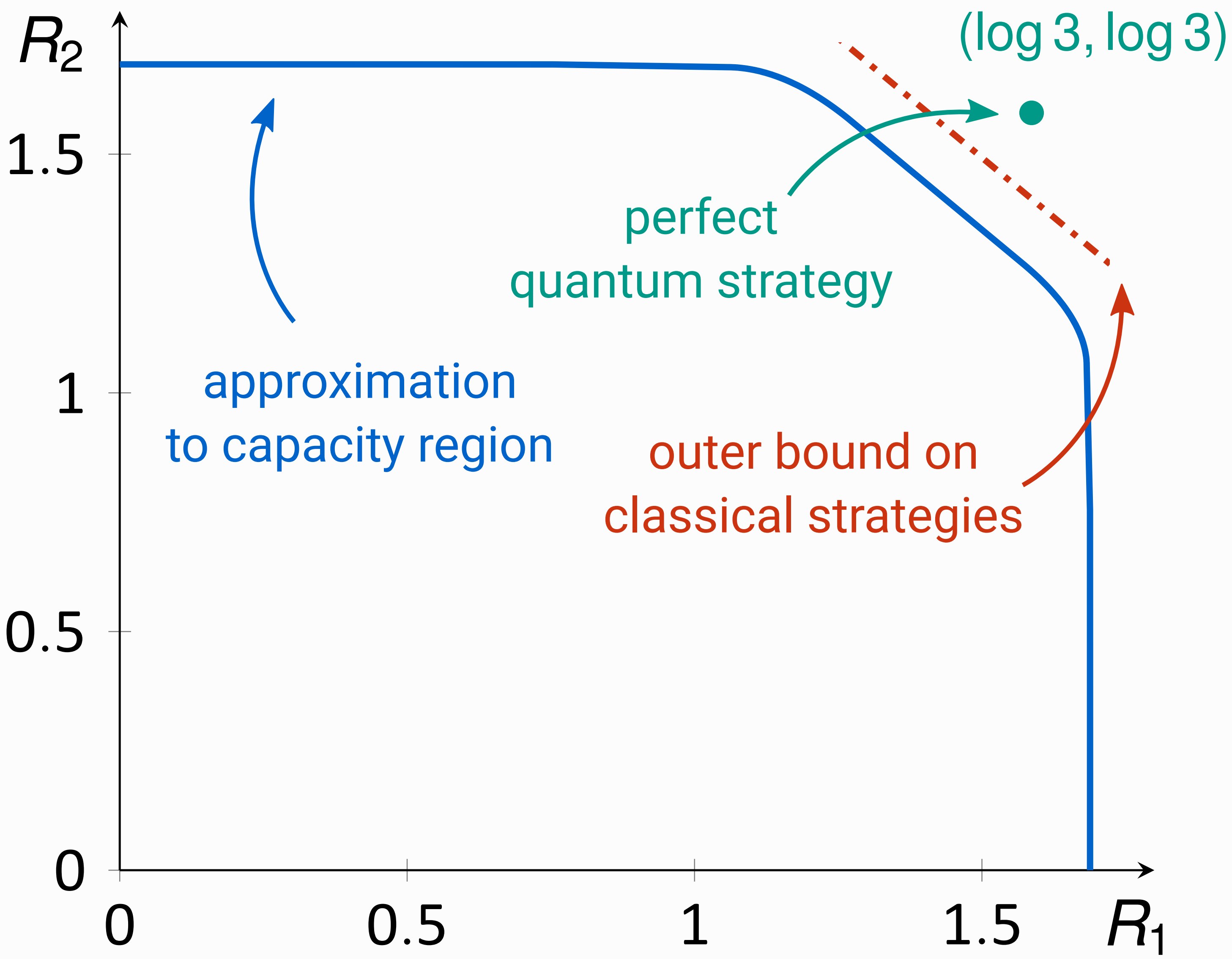
How can we **quantify** this different behavior?



Overview of main results: two angles

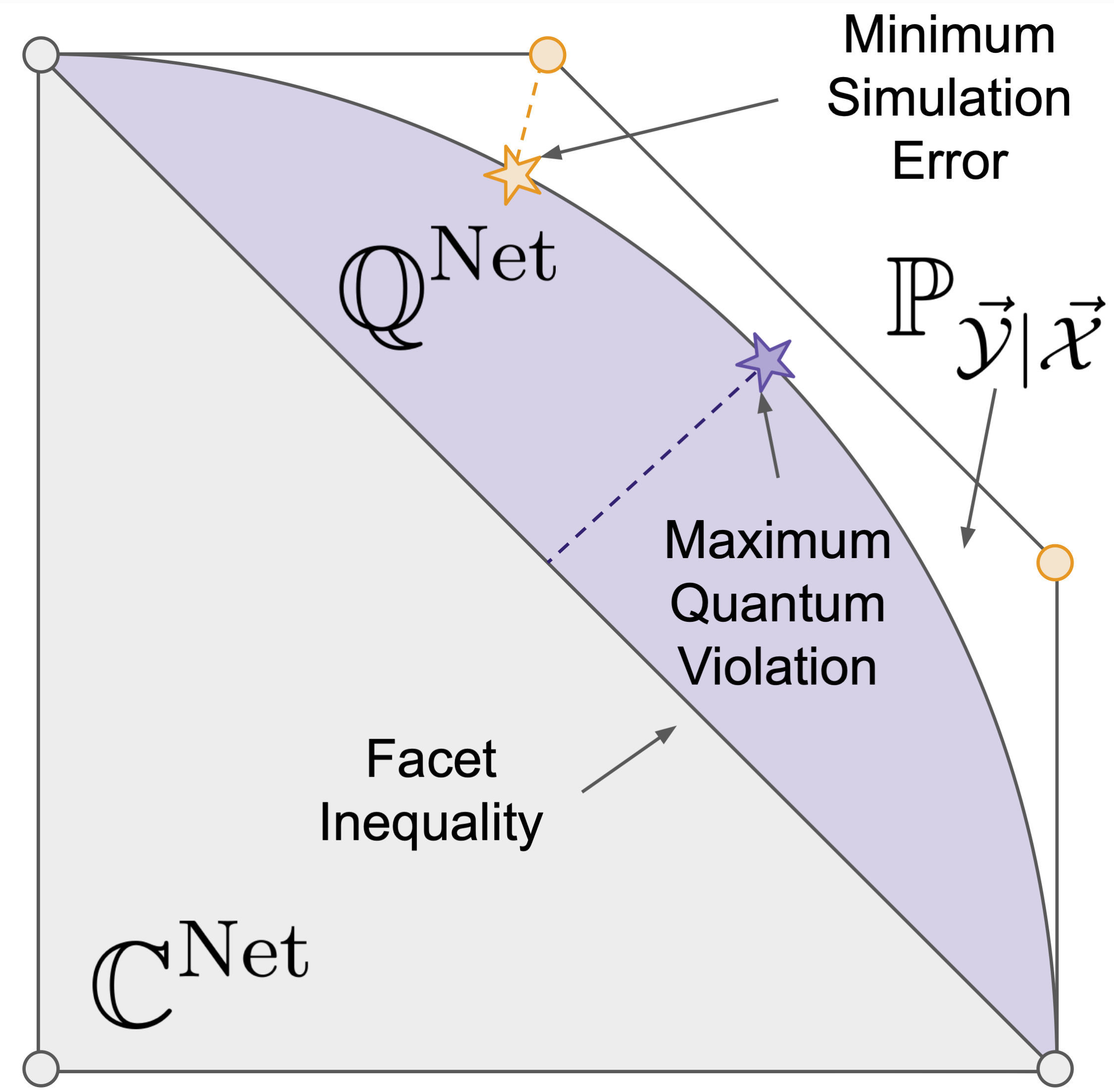
Information-theoretic:

Shared entanglement enhances capacity region of multiple access channels.



Operational nonlocality:

Network statistics violating facet inequalities bounding "classical polytope" certify genuinely quantum resources in an operational way.

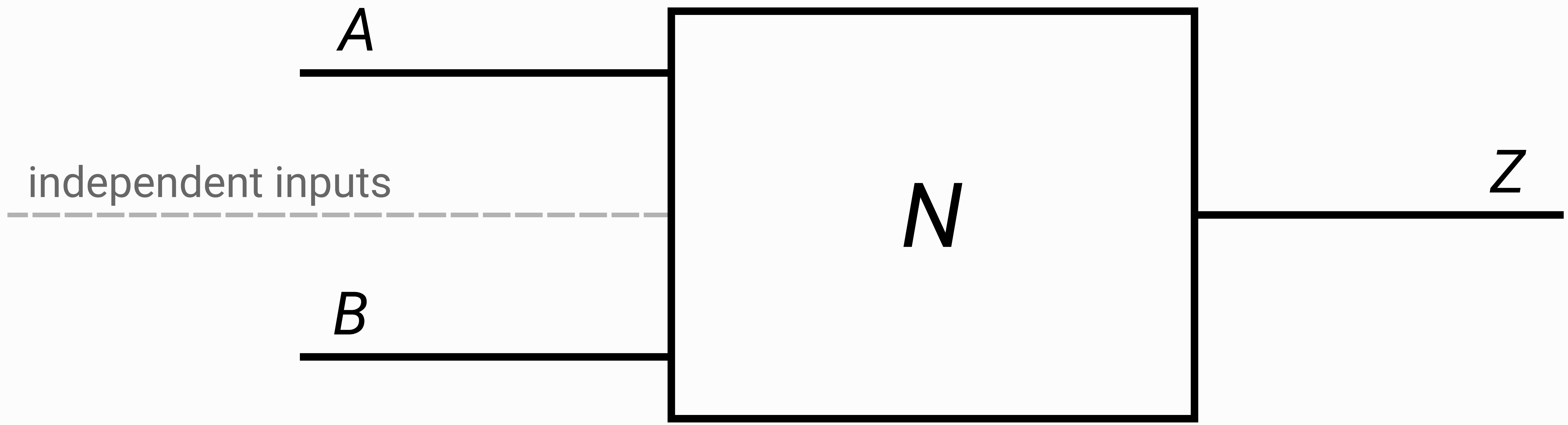


Multiple access channel

Two (or more) senders send messages to a **single receiver**.

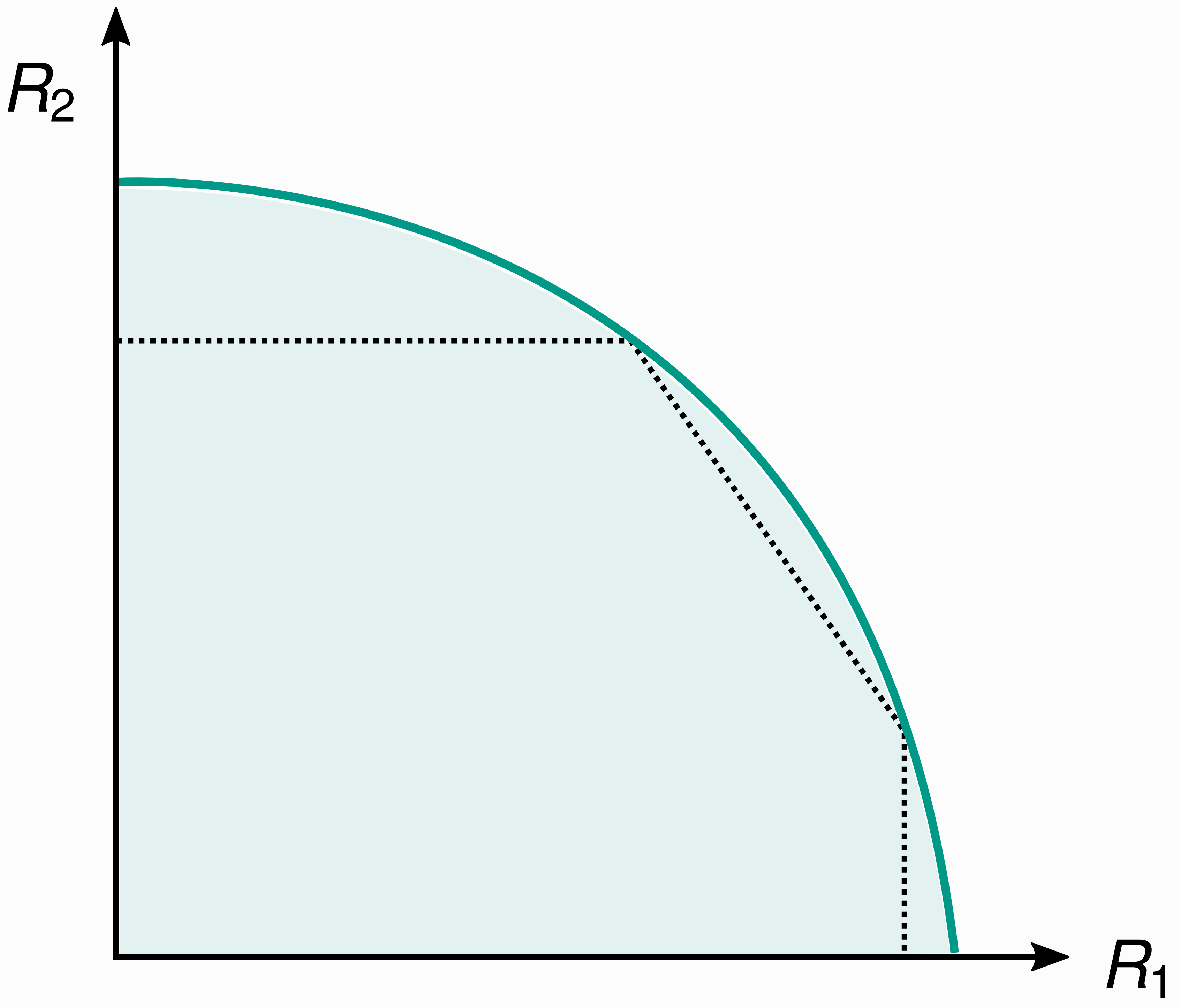
Mathematical model: conditional probability distribution $N(z|a, b)$.

Input RVs A and B are independent, channel N defines output RV Z.



Capacity region

Capacity region is defined as closure of all achievable rate pairs (R_1, R_2).



Coding theorem

Achievable rate pairs (R_1, R_2) satisfy

$$R_1 \leq \mathcal{I}(A; Z|B)$$

$$R_2 \leq \mathcal{I}(B; Z|A)$$

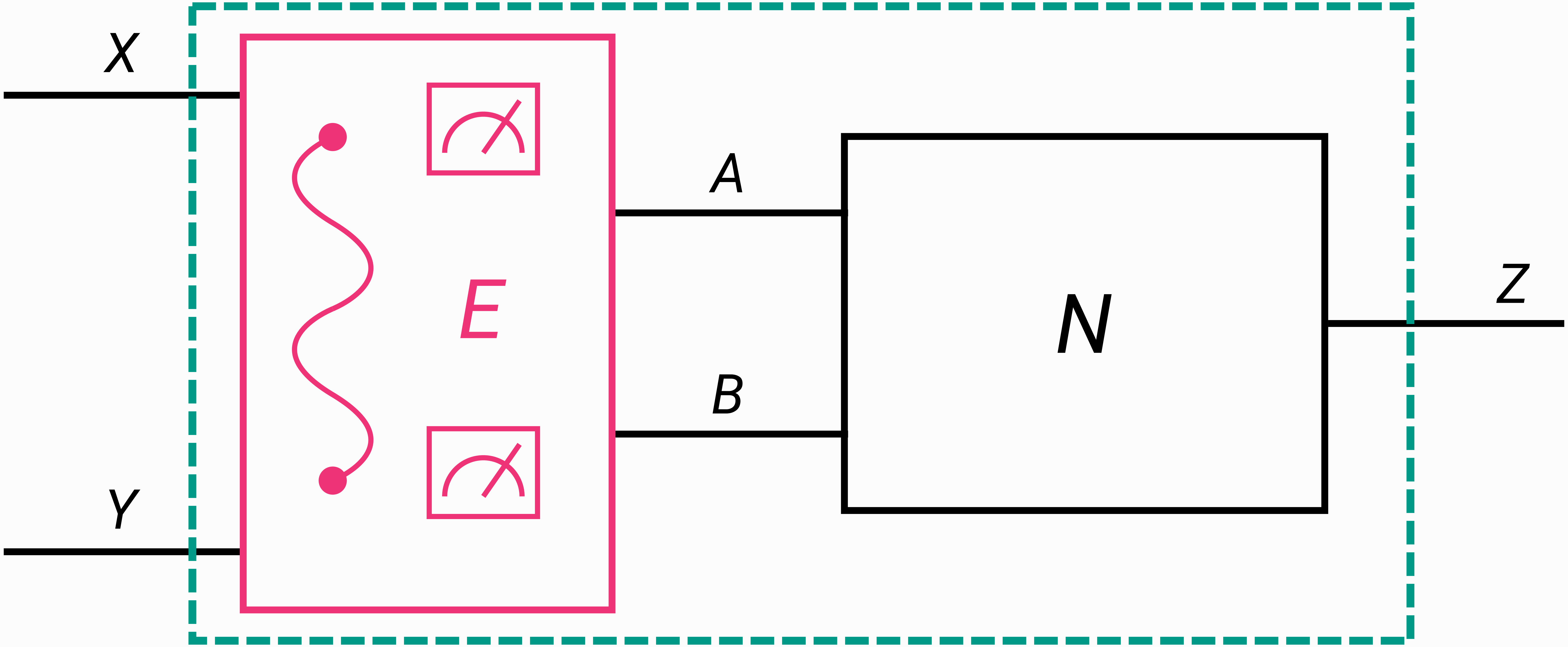
$$R_1 + R_2 \leq \mathcal{I}(AB; Z)$$

for any product distribution $p_{AB} = p_A p_B$.

[Ahlswede '71], [Liao '72]

Entanglement assistance

Effective MAC: $M(z|x, y) = \sum_{a,b} N(z|a, b)E(a, b|x, y)$

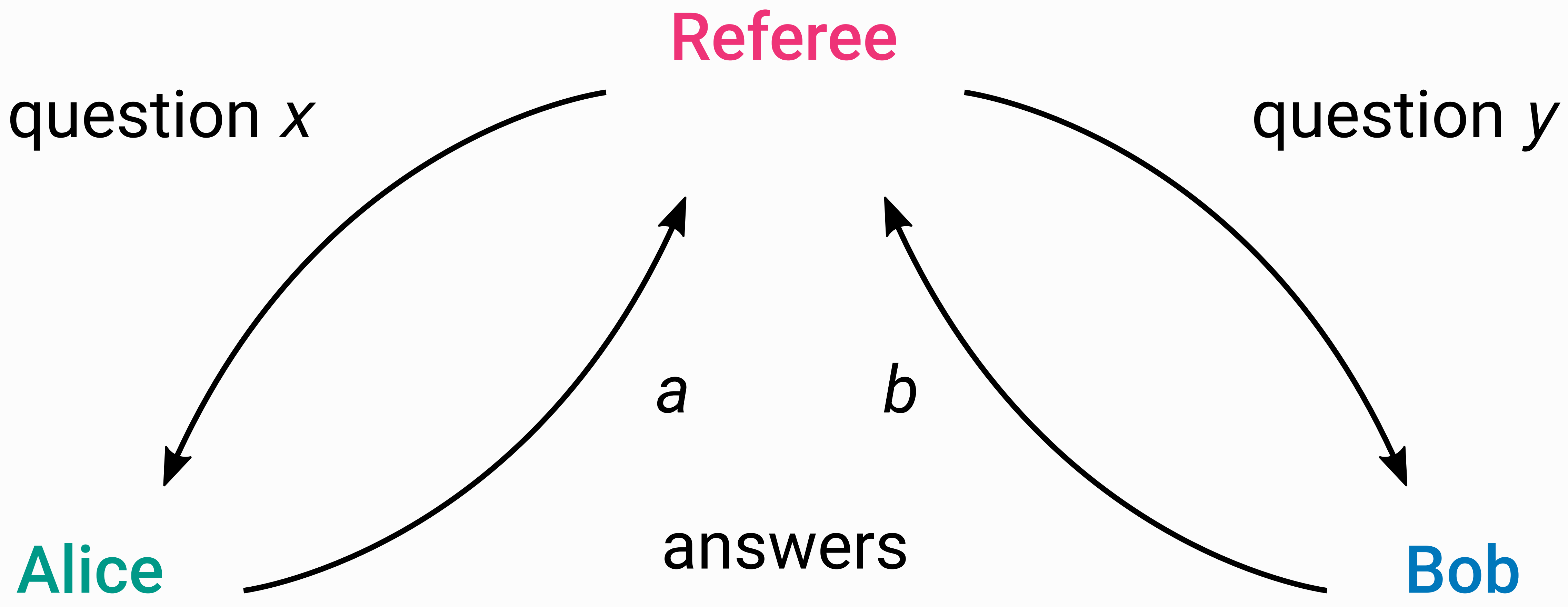


Non-local games

Alice and Bob answer questions from a referee **without any communication**.

They agree on a (classical, quantum, ...) strategy beforehand.

Alice and Bob win if question-answer tuple (x, y, a, b) lies in the **winning condition** W.



Example: CHSH game

Classical value $\omega(G)$: best winning probability with classical strategies.

Quantum value $\omega^*(G)$: best winning probability with quantum strategies.

CHSH game

Alice and Bob win if $a \oplus b = x \wedge y$.

$$0.75 = \omega(G_{\text{CHSH}}) < 0.85 = \omega^*(G_{\text{CHSH}}) < 1$$

Classical value: $\omega(G_{\text{CHSH}}) = 3/4$

Quantum value: $\omega^*(G_{\text{CHSH}}) = \cos^2(\pi/8) \approx 0.85$

Achieved by jointly measuring local shares of an ebit $|\Phi^+\rangle$ in one of two bases depending on question values x, y.

Example: Magic square game

0	0	0
0	1	1
1	0	?

$\omega(G_{\mathrm{MS}}) = 8/9$

Winning conditions

- Alice: row parity even
- Bob: column parity odd
- Same answer in overlapping cell

I Z	Z I	Z Z
X I	I X	X X
X Z	Z X	Y Y

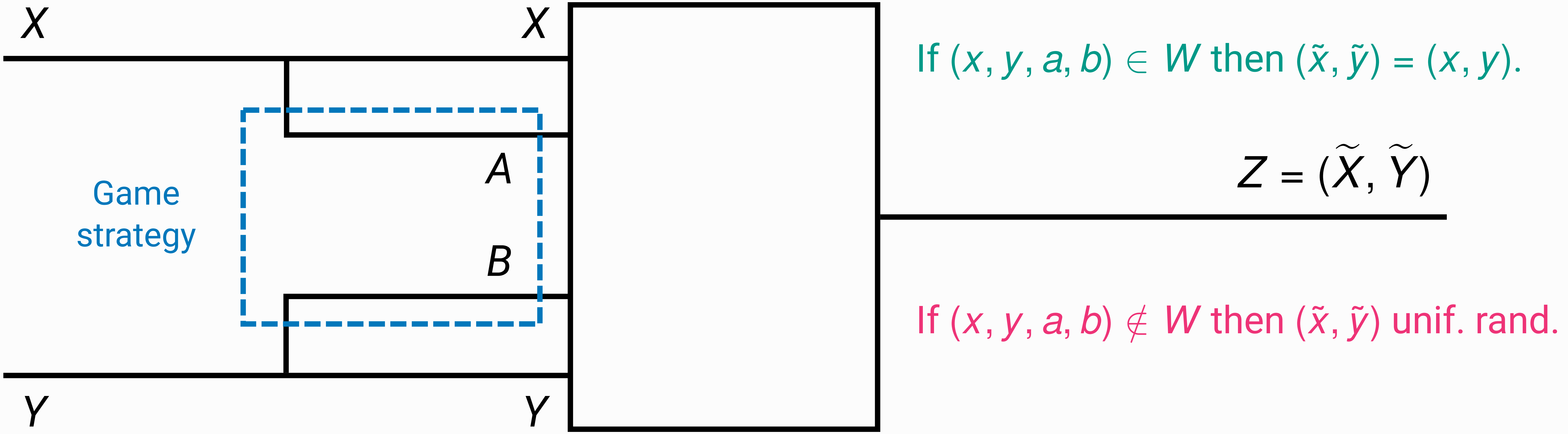
$\omega^*(G_{\mathrm{MS}}) = 1$

$$8/9 = \omega(G_{\mathrm{MS}}) < \omega^*(G_{\mathrm{MS}}) = 1$$

MAC based on non-local game

Idea: Senders play a non-local game G with each other and try to send the questions.

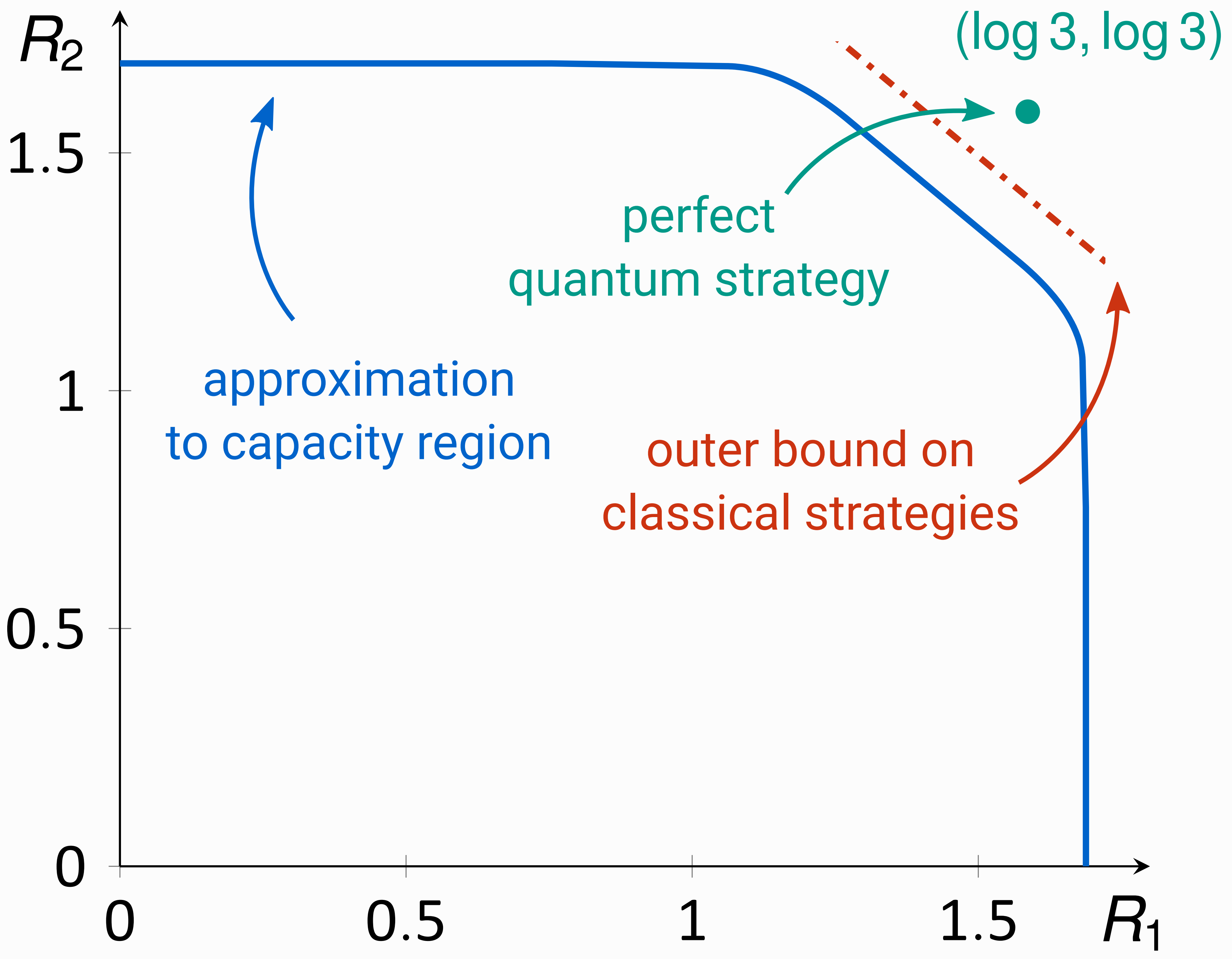
MAC N_G transmits questions noiselessly iff they win the game.



[Quek & Shor '17]

Enlarged capacity region

Let N_{MS} be the MAC defined in terms of the magic square game ($|Z| = 9, \omega(G) = 8/9$).



Outer bound on classical strategies:

$$R_1 + R_2 \leq \log\left(|Z| - 1 + |Z|^{-|Z|(1-\omega(G))}\right)$$

$$\approx 3.02$$

Using perfect quantum strategy:

$(R_1, R_2) = (\log 3, \log 3)$ is achievable,

for which $R_1 + R_2 = 2\log 3 \approx 3.17$.

Further results

There is a linear system non-local game for which a perfect quantum strategy needs unbounded entanglement. [Slofstra, Vidick '18], [Slofstra '19]

Main result: Top-right corner in the capacity region of the corresponding MAC can only be achieved in the limit of unbounded entanglement.

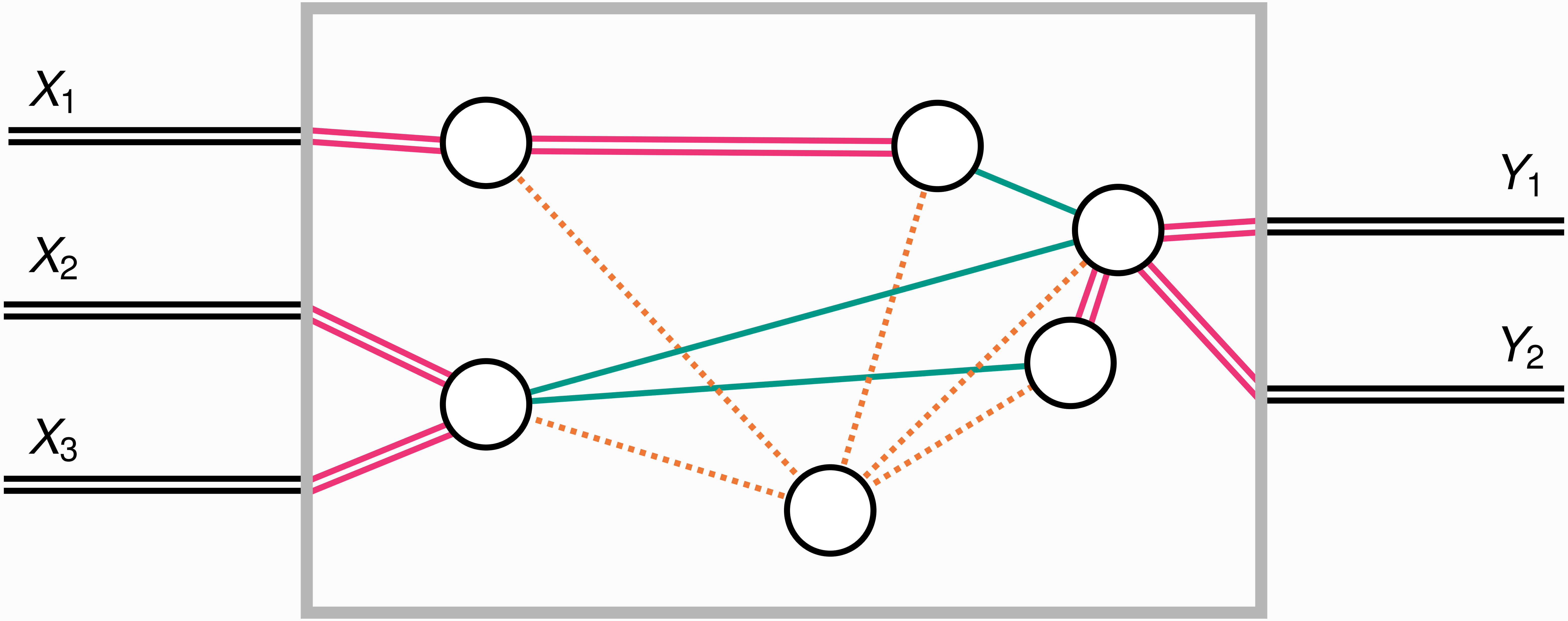
There is a non-local game version of 3SAT, for which it is NP-hard to decide whether there exists a perfect strategy. [Håstad '01]

Main result: It is NP-hard to decide for the corresponding MAC whether the top-right corner in the capacity region can be achieved.

Follow-up work: Noetzel '20, Pereg et al. '23, Yun et al. '23, ...

Certifying quantum resources in networks

Question: How do we certify that a communication network includes **genuinely quantum resources**?

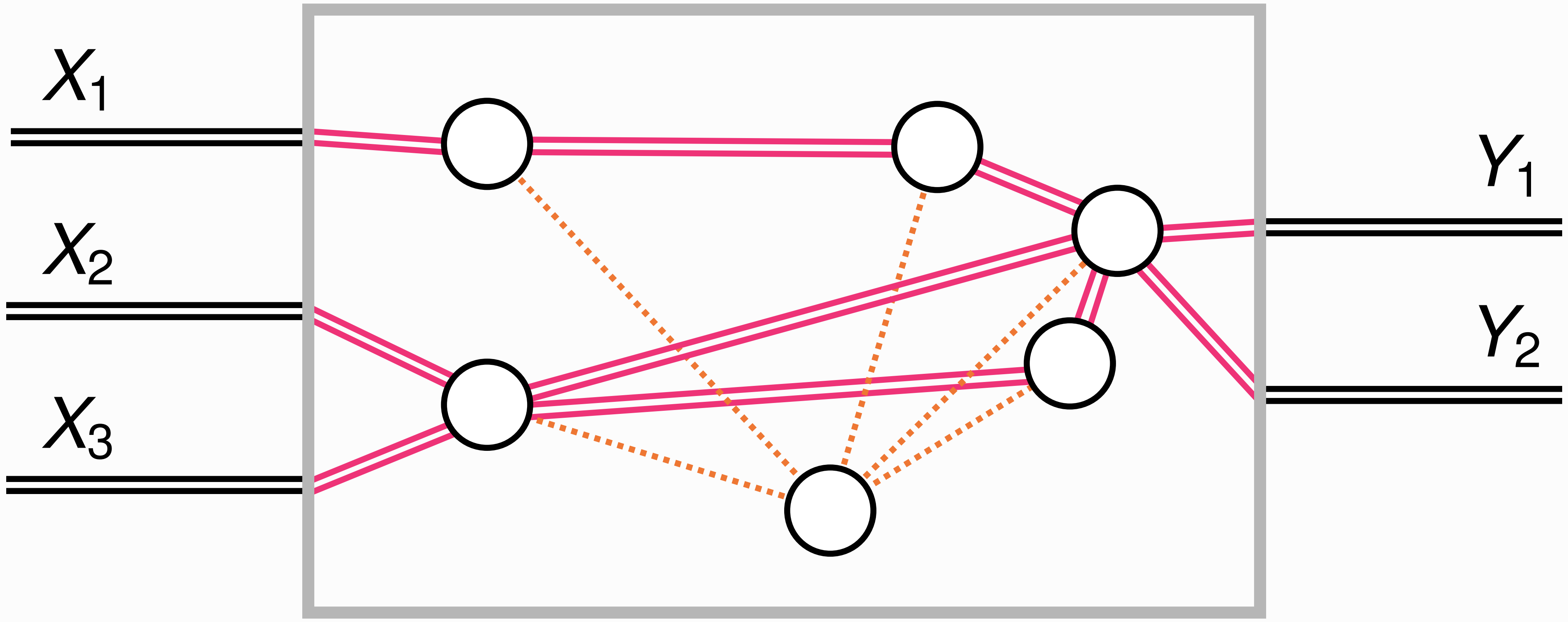


Model assumptions

Classical networks

Bounded classical communication at each node.

Global shared randomness available at every node.



Behaviour ($\equiv$ I/O distribution) $P(\vec{y}|\vec{x})$ is determined by communication network.

[Bowles et al. '15], [Frenkel & Weiner '15], [Doolittle & Chitambar '21]

Facet inequalities and classical simulation cost

Classical behaviors form **convex polytope** that can be written as intersection of half-spaces determined by **facet inequalities *F***:

$$\gamma \geq \langle F, P \rangle$$

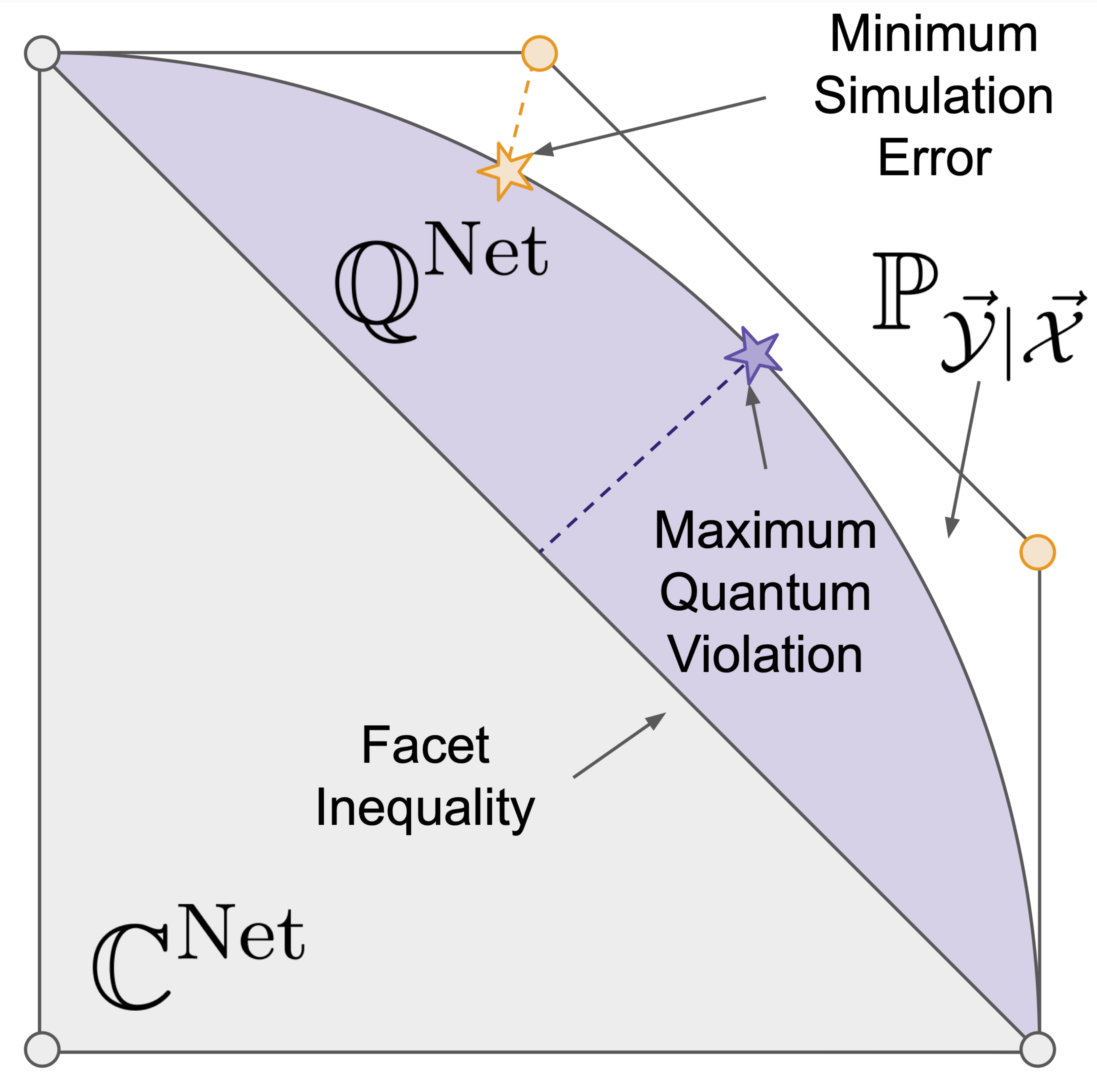
Behaviors *P* outside the classical polytope certify quantum resources.
For those behaviors we can **quantify non-classicality** using **classical simulation error**:

$$D(V, P) = \frac{1}{2|\vec{\mathcal{X}}|} \sum_{\vec{x},\vec{y}} |V(\vec{y}|\vec{x}) - P(\vec{y}|\vec{x})| = 1 - \frac{1}{|\vec{\mathcal{X}}|} \langle V, P \rangle$$

where *V* is a vertex (deterministic behavior) of the classical polytope.

[Bowles et al. '15], [Doolittle & Chitambar '21]

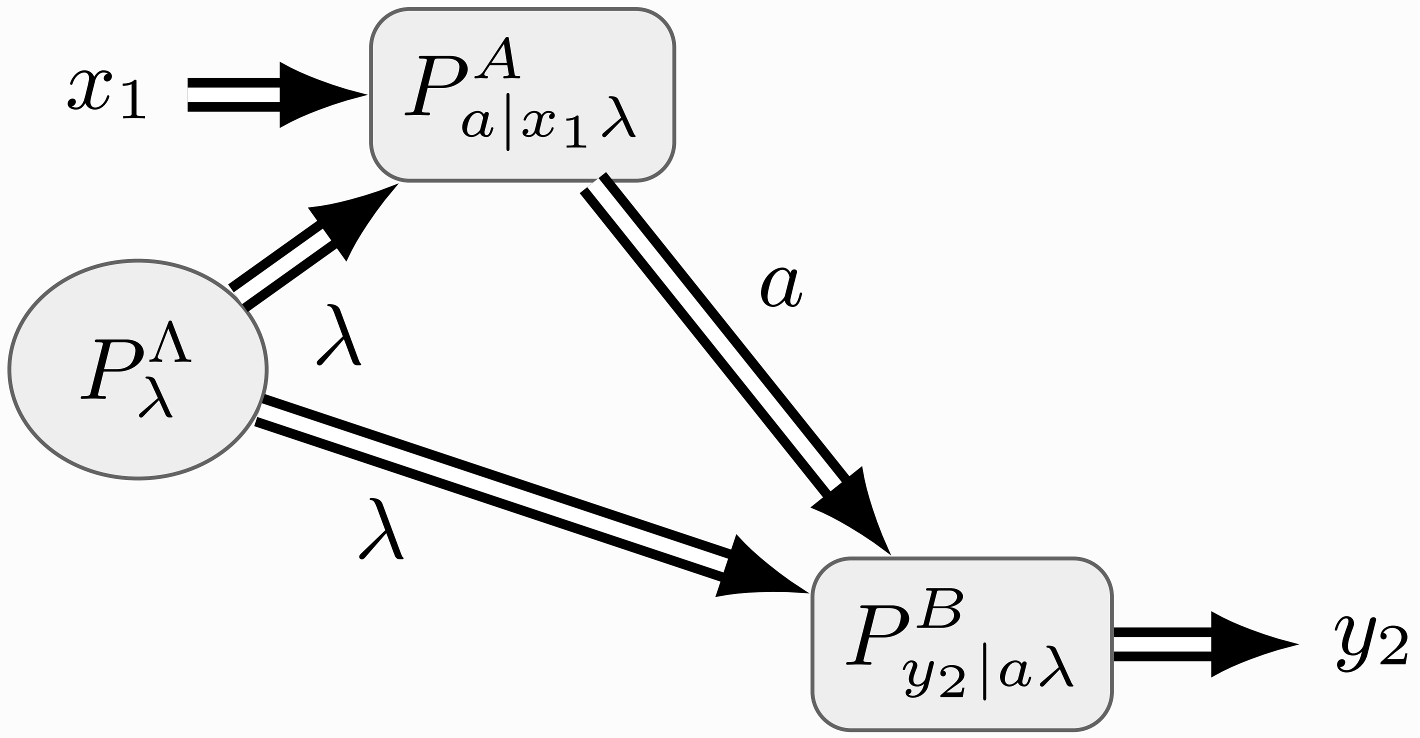
Classical polytope and quantum violations



Simple example: Point-to-point signaling

$x_1 \in [6], a \in [2], x_2 \in [4]$

Unbounded global shared randomness.



Every classical behavior P satisfies

$$\langle F_i, P \rangle \leq \gamma.$$

Facet inequalities (γ, F_i):

(a) $2 \geq \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $2 \geq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $3 \geq \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $4 \geq \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

(e) $4 \geq \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

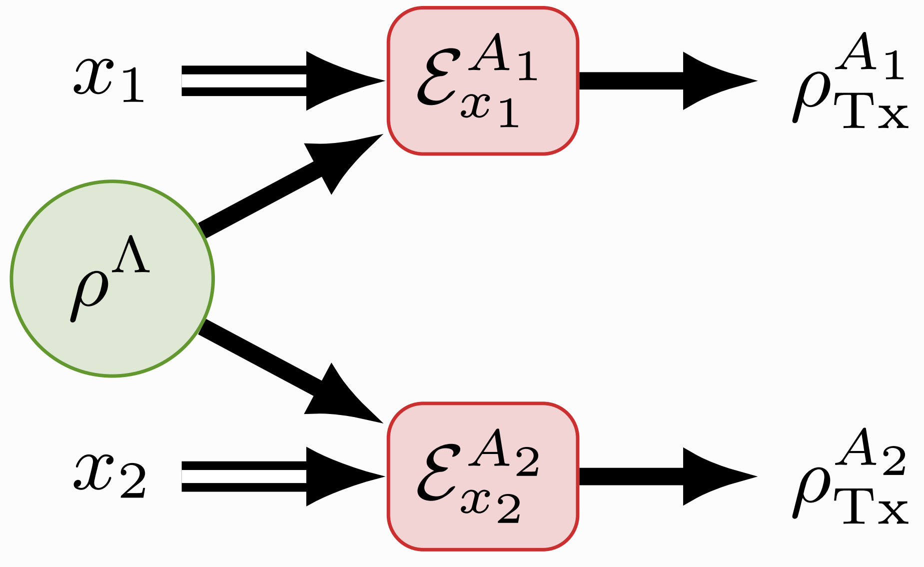
(f) $4 \geq \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

(g) $4 \geq \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$

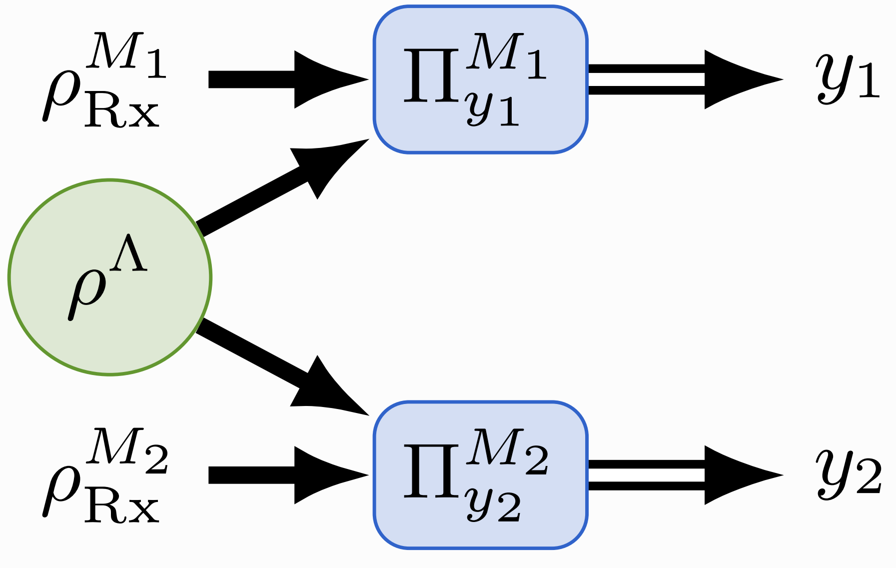
(h) $5 \geq \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

Quantum resources

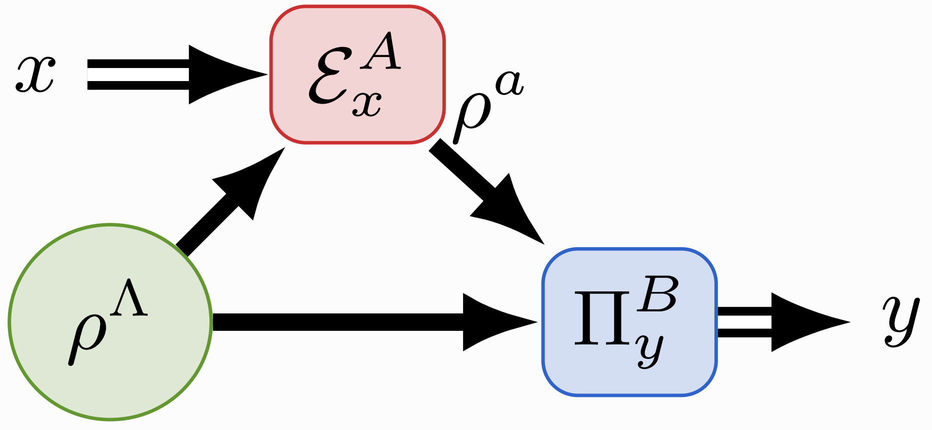
Quantum communication networks may include **shared entanglement**, **quantum communication**, and **measurements**.



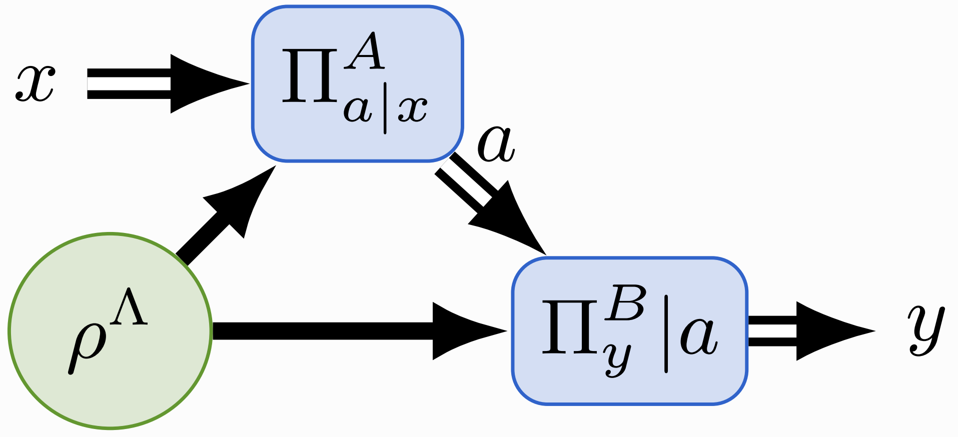
Entanglement-assisted senders



Entanglement-assisted receivers



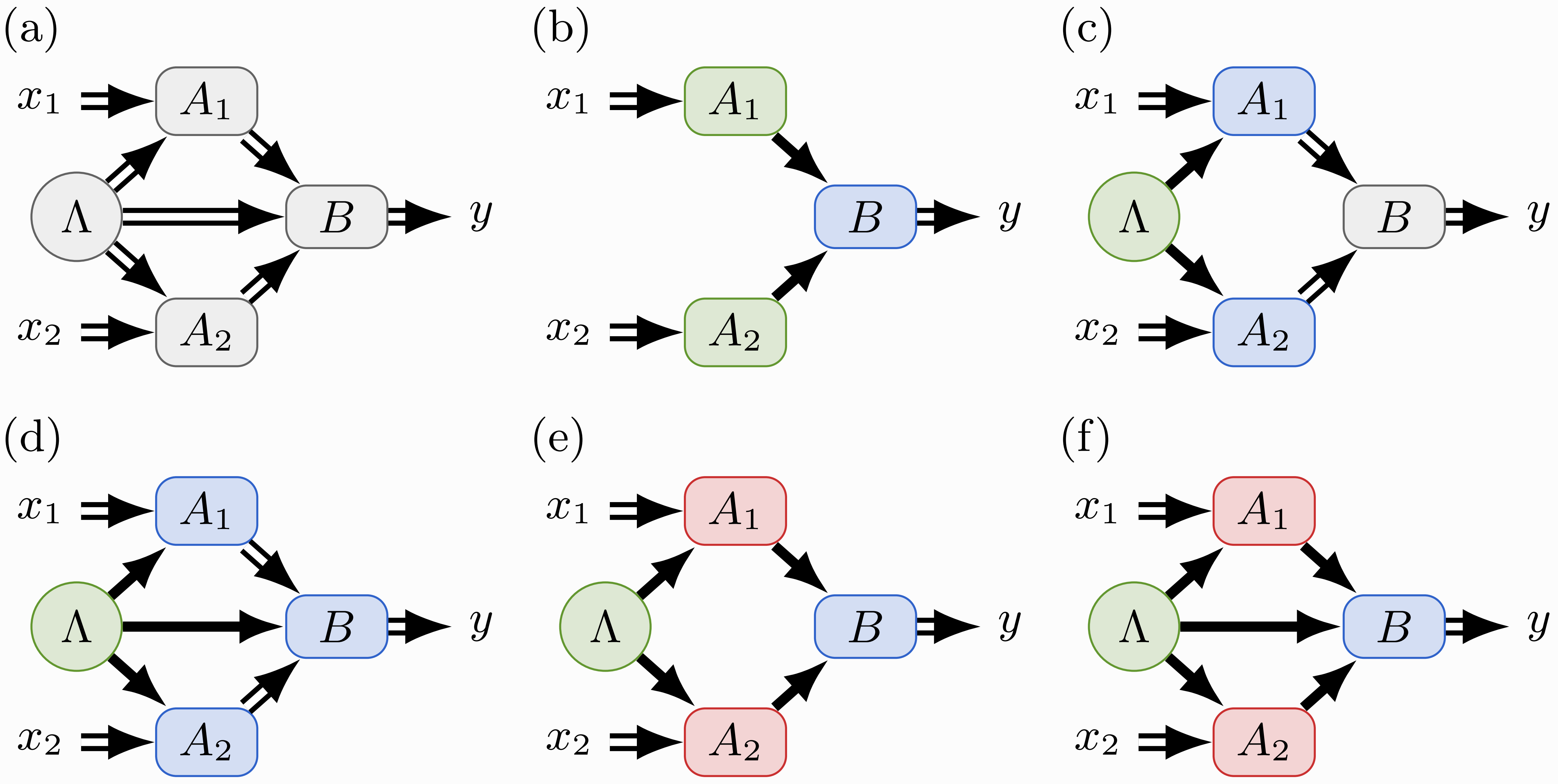
Entanglement-assisted
quantum communication

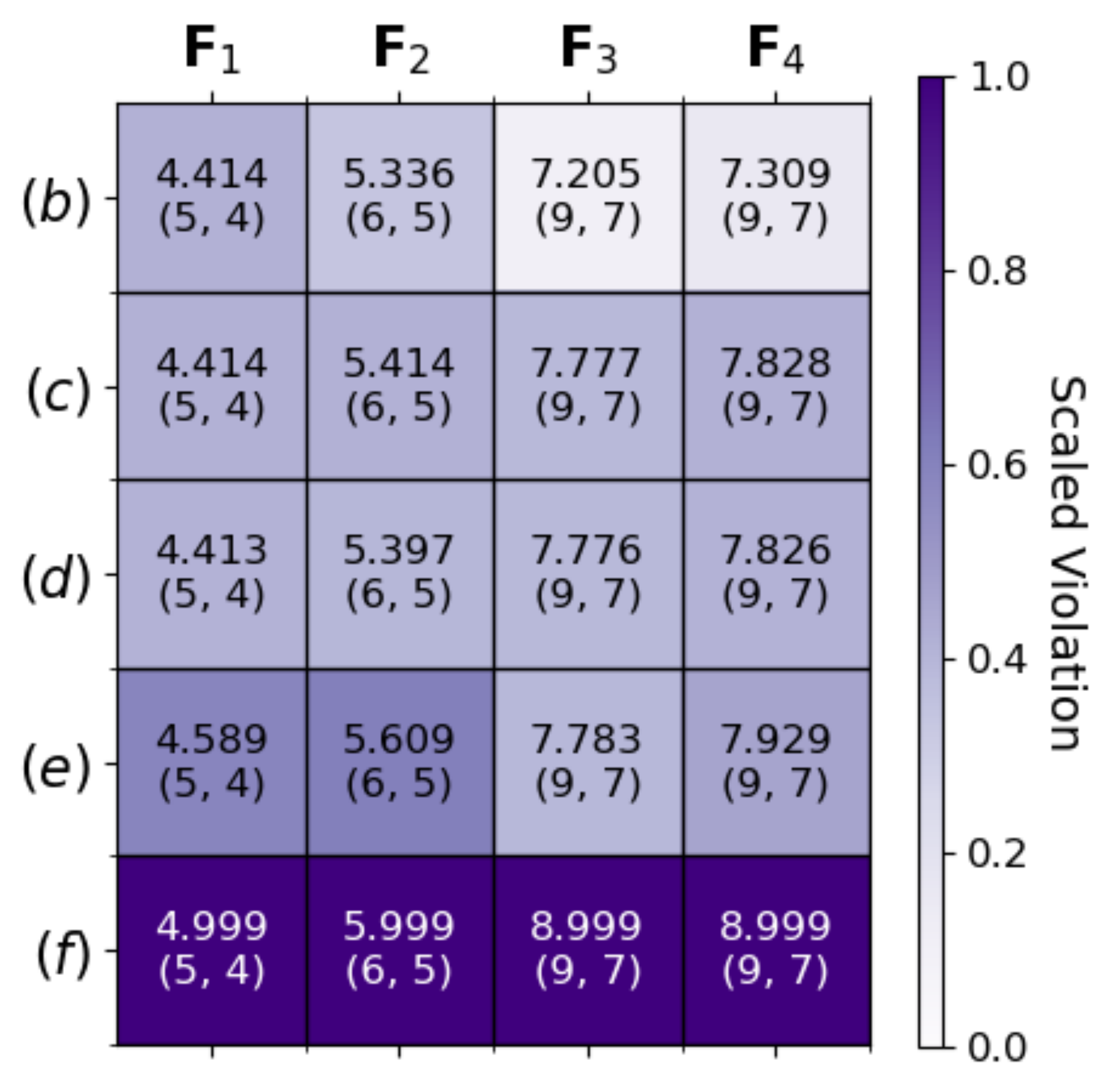


Entanglement-assisted
classical communication

Certifying non-classicality in networks

Main result: We use a variational quantum optimization ansatz to certify quantum resources in various network topologies using facet inequality violations.





Conclusion

Communication networks show **fundamentally different behavior** when equipped with **quantum resources** such as shared entanglement or quantum communication.

In a simple **multiple acccess channel** scenario, **shared entanglement** between senders may **increase the capacity region**, and we may need unbounded entanglement to achieve this advantage.

In arbitrary communication networks **facet inequalities** bound the polytope of all classical behaviors, and **violations** of these inequalities **certify quantum resources**.

Thank you for your attention!