Variational Quantum Optimization of Teleportation Protocols with Noisy Resources

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BACKGROUND

Teleportation is a key task in quantum communication that enables the transfer of quantum information between distant locations. It works by using a pre-shared entangled resource and classical communication to teleport an unknown quantum state from Alice to Bob.

The standard teleportation protocol [Bennett et al., Phys. Rev. Lett. 70, 1895, 1993] simulates a qubit channel with pre-shared entanglement and a two-bit classical channel. On the flip side, the **dense coding protocol** uses a noiseless qubit channel and entanglement to simulate a two-bit classical channel. We note that this duality extends to the level of the protocols [Chitambar, Leditzky, arXiv preprint, 2023].

A general teleportation protocol from Alice to Bob consists of:

- An entangled state, ρ_{ab}
- Encoding measurement, $\{\Pi^i\}$
- Decoding state transformation $\{D^i\}$

The same elements can be used to define a dense coding protocol from Bob to Alice where the state transformation encodes and the measurement decodes the classical information.

AIM

Our research focuses on finding good teleportation protocols for noisy quantum resources.

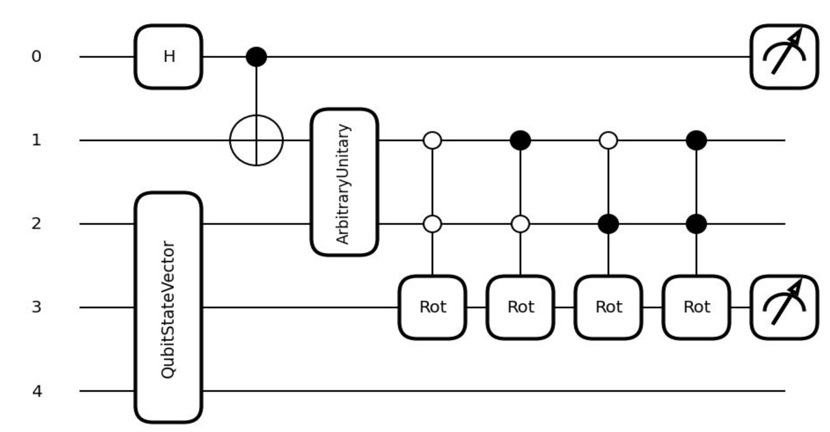
- A good teleportation protocol is one in which the teleported state approximates the original state as much as possible.
- A noisy quantum resource is a low-quality entangled resource.

While the standard teleportation protocol gives a perfect protocol for noiseless resources, one aims to maximize the teleportation fidelity of a protocol in the noisy setting by finding suitable measurements and correction operations.

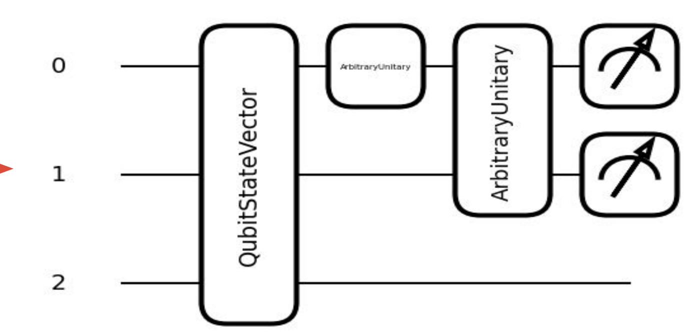
Our study uses the correspondence between teleportation and dense coding to efficiently optimize over teleportation protocols. We focus on protocols using mixtures of two weighted Bell states as the entanglement resource.

QNETVO ANSATZ

We use Pennylane (https://pennylane.ai/) and the QNETVO software package (https://github.com/ChitambarLab/qNetVO) to implement our optimization strategy. Specifically, we are parameterizing operators and using gradient descent to maximize the fidelity of the protocol. Our coding framework, QNETVO, provides a way to describe a protocol in a quantum network and optimize over it.



A parameterized teleportation protocol with a noisy pre-shared entangled resource between Alice and Bob.

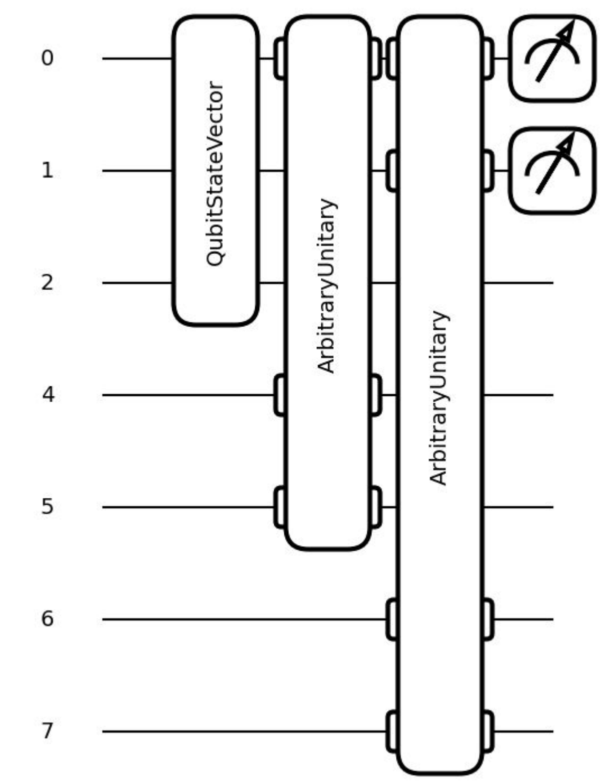


A parameterized dense coding protocol that needs fewer quantum resources and can hence be optimized more efficiently.

In order to optimize over more general measurements (given by a POVM) and noisy decoding operations, we introduce additional wires that enable us to describe mixed states that are purified by the extra qubits.

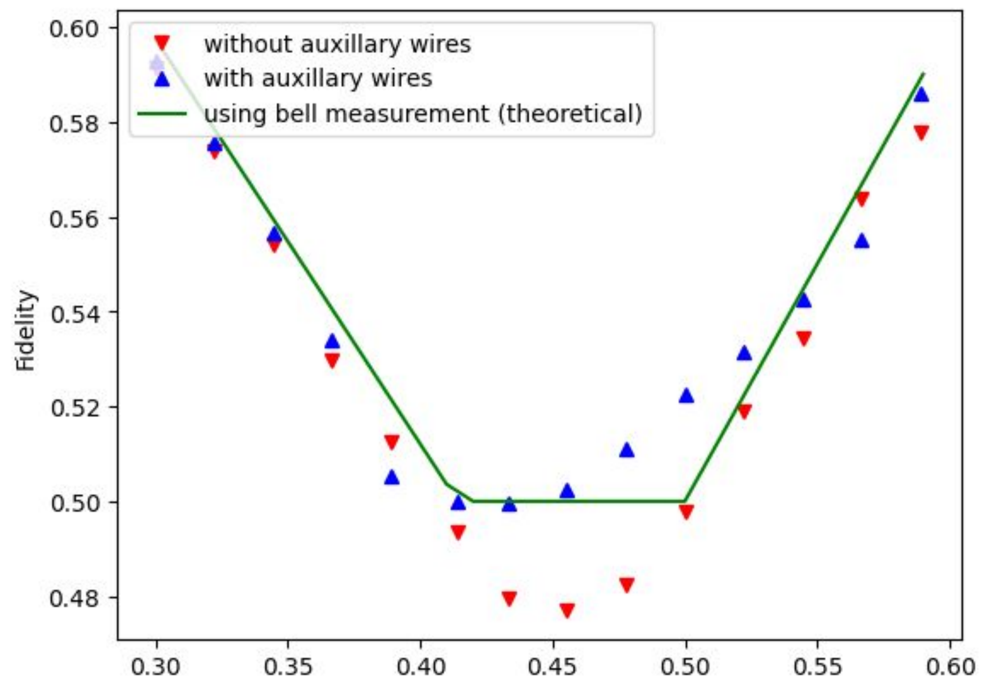
Although we have a fully general decoding operation, our measurement requires two more auxiliary qubits to achieve full generality. As the number of auxiliary wires increases, the number of parameters in the circuit grows exponentially.

Therefore, we set a cutoff as indicated on the right side, as we can achieve fairly good results without requiring more advanced equipment than a standard laptop. In this case, the number of parameters is 318, which is much lower than the 4158 required for the fully general solution.

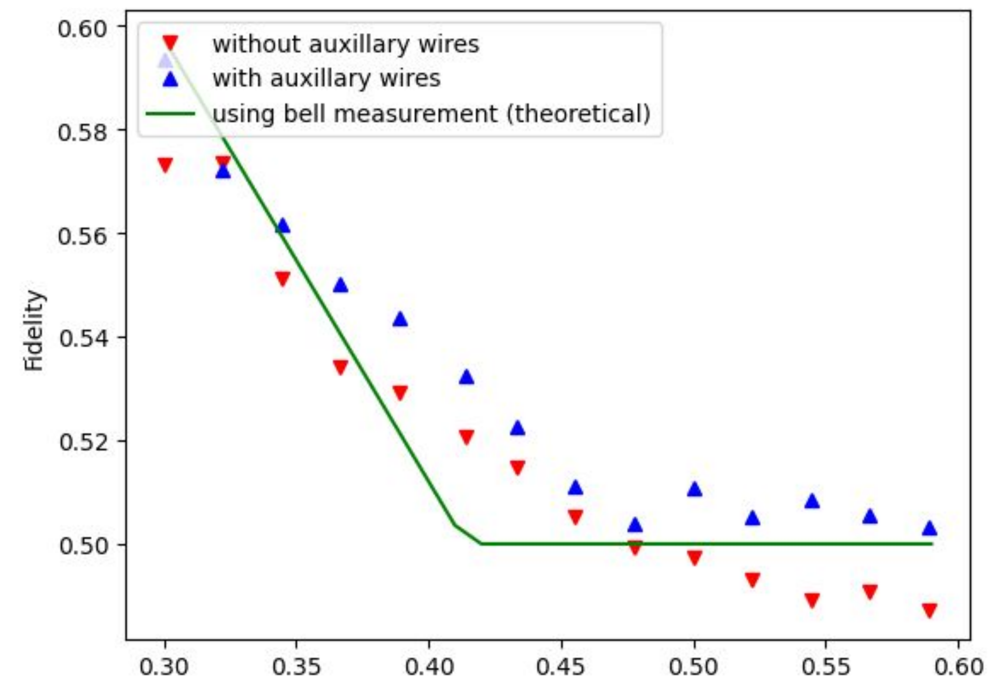


RESULTS

Consider the pure states $|\Psi_1\rangle = (x)^{1/2}\,|00\rangle - (1-x)^{1/2}\,|11\rangle$ and $|\Psi_2\rangle = (y)^{1/2}\,|01\rangle - (1-y)^{1/2}\,|10\rangle$.
Our resource state is $\rho_{ab} = p|\Psi_1\rangle\langle\Psi_1| + (1-p)|\Psi_2\rangle\langle\Psi_2|$ (see also [Badziag et al., PRA 62, 012311, 2000]).



*This plot for x=0, y=(3-2*2^{1/2})/(4-2*2^{1/2})) reveals around p=0.5 that we can surpass the optimal theoretical fidelity of a protocol using a Bell measurement with our optimized noisy ansatz.*



This plot for x=1/2 and y as in the left plot reveals around p=0.41 that we can surpass the optimal fidelity of a protocol using a Bell measurement with both optimized noisy and noiseless ansätze.

COST FUNCTION

The Fidelity F of a teleportation protocol is a measure of how well the teleported state matches the original state, and we would like to maximize this quantity.

Let $(\rho_{ab},\{\Pi_i\},\{D_i\})$ be a $|C|$-dimensional teleportation protocol. Then the following relation holds:

$$F = |C|^{-2} \sum \mathrm{tr}(\Pi^i_{AC} w^i_{AC}) = N\,|C|^{-2}\, p_{succ}$$

where p_{succ} is the success probability of the POVM $\{\Pi_i\}$ discriminating the states w^i_{AC} drawn uniformly at random.

In our case, $N = 4 = |C|^2$, and hence the success probability p_{succ} in the dense coding protocol is equal to the fidelity in the corresponding teleportation protocol.

CONCLUSIONS

Our results demonstrate that our optimized noisy and noiseless ansatz can surpass the optimal theoretical fidelity of a protocol using a Bell measurement. Moreover, our approach efficiently optimizes a teleportation protocol using noisy resources, exploiting the duality between teleportation and dense coding.

ACKNOWLEDGEMENTS

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